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BASIC RESEARCH AND DATA ANALYSIS FOR THE NATIONAL GEODETIC SATELLITE PROGRAM AND FOR THE EARTH SURVEYS PROGRAM

Tenth Semiannual Status Report

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PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, OSU, and it is under the technical direction of Messrs. Jerome D. Rosenberg, Deputy Director, Communications Programs, OSSA and Benjamin Milwitzky, Deputy Director, Special Programs, Office of Applications, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C. 20546.

TABLE OF CONTENTS

1. STATEMENT OF WORK

The statement of work for this project includes data analysis and supporting research in connection with the following broad objectives:

(1) Provide a precise and accurate geometric description of the earth's surface.

(2) Provide a precise and accurate mathematical description of the earth's gravitational field.

(3) Determine time variations of the geometry of the ocean surface, the solid earth, the gravity field, and other geophysical parameters.

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2. ACCOMPLISHMENTS DURING THE REPORT PERIOD

2.1 Adjustment of the BC-4 Worldwide Geometric Satellite Triangulation Net

2.11 Theoretical Developments

As was mentioned in the last semi-annual report, work was begun on processing the NOAA TYPE II data. The only existing computer program at that time was designed to use non-correlated data. The TYPE II data, being the result of a polynomial fit to plate images, has an associated 14 x 14 variance-covariance matrix, and in order to use this data it was necessary to write a new program.

The new linear form of the mathematical model is

$$\begin{aligned} F_1 &= X_S - X_G - R\cos\alpha\cos\delta \\ F_2 &= Y_S - Y_G - R\sin\alpha\cos\delta \\ F_3 &= Z_S - Z_G - R\cos\delta \end{aligned} \tag{1}$$

where the subscripts S and G refer to satellite and ground, respectively, and R is the range from the ground station to the satellite. The observations are α and δ.

The linearized form of the mathematical model is basically the same as described in The Ohio State University, Department of Geodetic Science Report No. 86, (pp. 21-27), which is

$$AX + BV + W = 0, \tag{2}$$

where the matrices A and B are the partial derivatives with respect to the parameters and the observations, respectively. Whenever a satellite event is defined as the observations to one satellite position, the A matrix for one ground station and one satellite position is of the form

$$A = \left[\begin{array}{ccc|ccc} +1 & 0 & 0 & -1 & 0 & 0 \\ 0 & +1 & 0 & 0 & -1 & 0 \\ 0 & 0 & +1 & 0 & 0 & -1 \end{array}\right] = [+I \mid -I] \tag{3}$$

However, in case of correlated observations an event is defined as all observations to the seven (7) satellite positions, and the A matrix for one ground station and seven satellite positions takes the form

$$A = \left[\begin{array}{c|c} -I & \\ -I & \\ -I & \\ -I & I \\ -I & \\ -I & \\ -I & \end{array}\right]_{(21 \times 24)} . \tag{4}$$

This is perhaps easier to understand if the linearized form of the mathematical model is split up as follows:

$$A_1X_G + A_2X_S + BV + W = 0. \tag{5}$$

This is essentially what was done in the original adjustment program. But when the model in the original program is split up, the A matrices are either +I or -I and they cancel out in the mathematical development, the only change being that of signs. For the correlated data, A_1 is the left side of equation (3), and it will not cancel out.

Another change that had to be made was in the formation of the matrix

$$M^{-1} = (BP^{-1}B')^{-1}. \tag{6}$$

The problem arises because $BP^{-1}B'$ is a singular matrix and cannot be inverted. For the case of one ground station and one satellite position one can use the following

$$M^{-1} = (BP^{-1}B')^{-1} = (B')^{-1}P(B)^{-1} = (B^{-1})'\,PB^{-1}, \tag{7}$$

where

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial \delta} & \frac{\partial F_1}{\partial R} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial \delta} & \frac{\partial F_2}{\partial R} \\ \frac{\partial F_3}{\partial \alpha} & \frac{\partial F_3}{\partial \delta} & \frac{\partial F_3}{\partial R} \end{bmatrix} \tag{8}$$

$$P = \begin{bmatrix} \begin{bmatrix} \sigma_\alpha^2 & \sigma_{\alpha\delta} \\ \sigma_{\alpha\delta} & \sigma_\delta^2 \end{bmatrix}^{-1} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{bmatrix} \tag{9}$$

As can be seen in equation (7), the matrix B must be inverted, which means that it must be square. Thus even though the range R in equation (1) is not an observed quantity, it must be considered as such in order to make B a square matrix. This is of course compensated by inserting zeroes in equation (9).

The above development for M^{-1} is described in the above mentioned Report. In case of correlated images the situation is somewhat more complicated. The matrix B is now of dimensions 21 x 21 and of the form

$$B = \begin{bmatrix} \square & & & & & & O \\ & \square & & & & & \\ & & \square & & & & \\ & & & \square & & & \\ & & & & \square & & \\ & & & & & \square & \\ O & & & & & & \square \end{bmatrix} \tag{10}$$

where each of the blocks is a 3 x 3 as defined in equation (8).

The matrix P cannot be defined quite as simply as in equation (9). The original variance-covariance matrix is 14 x 14, and the P matrix is 21 x 21. This is handled as follows:

$$
W = \begin{bmatrix}
\sigma_{a1}^2 & \sigma_{a\delta 1} & \cdots & \sigma_{a1\delta 7} \\
\sigma_{a1\delta 1} & \sigma_{\delta 1}^2 & & \vdots \\
\vdots & & \ddots & \vdots \\
 & & \sigma_{a7}^2 & \sigma_{a7\delta 7} \\
\sigma_{a1\delta 7} & \cdots & & \sigma_{\delta 7}^2
\end{bmatrix}_{(14 \times 14)}
=
\begin{bmatrix}
\omega_{11} & \omega_{12} & \cdots & \omega_{1,14} \\
\omega_{21} & \omega_{22} & \cdots & \omega_{2,14} \\
\vdots & & & \vdots \\
\omega_{14,1} & \cdots & & \omega_{14,14}
\end{bmatrix}
\tag{11}
$$

$$
P = \begin{bmatrix}
\omega_{11} & \omega_{12} & 0 & \omega_{13} & \omega_{14} & 0 & \cdots & \omega_{1,13} & \omega_{1,14} & 0 \\
\omega_{21} & \omega_{22} & 0 & \omega_{23} & \omega_{24} & 0 & \cdots & \omega_{2,13} & \omega_{2,14} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\omega_{31} & \omega_{32} & 0 & & & & & & & \\
\omega_{41} & \omega_{42} & 0 & & & & & & & \\
0 & 0 & 0 & & & & & & & \\
\vdots & & & & & & & & & \\
\omega_{13,1} & \omega_{13,2} & 0 & & \cdots & & & \omega_{13,13} & \omega_{13,14} & 0 \\
\omega_{14,1} & \omega_{14,2} & 0 & & \cdots & & & \omega_{14,13} & \omega_{14,14} & 0 \\
0 & 0 & 0 & & \cdots & & & 0 & 0 & 0
\end{bmatrix}_{(21 \times 21)}
\tag{12}
$$

By using the matrices B from (8) and P from (12), equation (7) can be solved for M^{-1} (the notation M^{-1} is a misnomer, but this expression was used in Report No. 86 and we have continued with the same notation). The complete description fo the mathematical will be given at a later date.

By using the techniques described abouve, the reduced normal equations are formed as described in Report No. 86.

In addition to the generalized approach described above, a completely different mathematical model has also been developed using the method of observation equations. The principal advantage of the method of observation equations is that here the original given correlation matrix is used without any modifications which is necessary in the generalized least squares approach.

2.12 Data Acquisition

As of the end of this reporting period the following BC-4 data has been received from the data center:

(i) Type I Data - 31 Tapes.
(ii) Type II Data - 15 Tapes.

The tape-wise details for type II data are as listed below:

Tape Serial No.	No. of events on the tape	Break up of events with simultaneously observing stations		
		2 stations	3 stations	4 stations
A-10806	87	73	12	2
A-10268	90	76	13	1
A-11082	90	70	17	3
A-03725	90	70	20	-
A-03719	90	74	14	2
A-03727	90	62	25	3
A-03728	90	68	20	2
A-10897	89	71	17	1
A-03738	30	19	11	-
A-95575	29	22	7	-
A-11519	60	40	20	-
A-12327	30	28	2	-
A-12037	60	55	5	-
A-12010	30	26	4	-
A-14094	60	47	13	-
	1015	801	200	14

2.2 Investigations Related to the Problem of Improving Existing Triangulation Systems by Means of Satellite Super-Control Points

2.21 Introduction

Geodetic triangulation has been accepted as an accurate method of determining "precise" coordinates for the triangulation stations of relatively short chains. This well-accepted idea was also given in an article "How accurate is First-Order Triangulation?" [Simmons, 1950, pp. 53-56] with the following introductory words:

> The question is often asked, "How accurate is the position of a triangulation station," or "To what accuracy are the distances between triangulation stations known?" These questions are difficult to answer, principally because first-order triangulation gives the optimum accuracy in the measurement of great distances and there is at present no super yardstick to which it can be compared.

Two modern technological advancements, namely, satellites and electronic distance measuring (EDM) instruments, have questioned the first-order triangulation accuracy, especially if triangulation is extended to distances longer than 1000 km or more. In such extended triangulation systems systematic errors like lateral refraction, propagation of observational errors, residual polar motion effects on latitude, longitude and azimuth, etc. [Mueller, 1969, pp. 61, 86-87; Pellinen, 1970, pp. 34-35; Wolf, 1950, pp. 117], which cannot be eliminated, accumulate. Lately the question has been raised whether any significant increment to accuracy is "cascaded" from a 1:1 million 1000 km net through a 100 km net to local control over 10 km distances.

The satellite triangulation and super-transcontinental traverse, being of the highest achievable accuracy of today, i.e., super-control net of "zeroth" order, constitute a modern geodetic super structure, within which the classical geodetic triangulation is supposed to provide a geodetic control densification.

According to the classical geodetic concept, a lower order system should be tied to a higher order system. Statistically, this means that the variance-covariance of the higher order system, as a lower limit for accuracy, be at least compatible with the internal precision of the lower order system. For all practical reasons, the accuracy of the higher order systems should be substantially better (by a factor of two to three) than the subordinated system, thus supplying a rigorous constraint in the reduction of the lower order system [Schmid, 1969, p. 4].

The objective of this investigation is to answer the question: Whether any significant increment to accuracy could be transferred from a super-control net to the basic geodetic net (first-order triangulation). This objective was accomplished by evaluating the positional accuracy improvement for station Wyola (95), which is near the middle of the investigated geodetic triangulation net, by using various station constraints over its geodetic position.

2.22 Data and Accuracy Estimates

For the purpose of the present investigation, the triangulation of the western-half of the United States has been considered, as this is more accurate than that of the eastern-half of the United States [Simmons, 1950, p. 54]. The investigation is done on the chain from Moses Lake, Washington to Chandler, Minnesote (Figure 1), as these two stations are also common on both the continental satellite net (CSN) and the super-transcontinental traverse (STT). The data used were supplied by the Triangulation Branch of Geodesy Division, and the Geodetic Research and Development Laboratory, both of the National Oceanic and Atmospheric Administration, Washington.

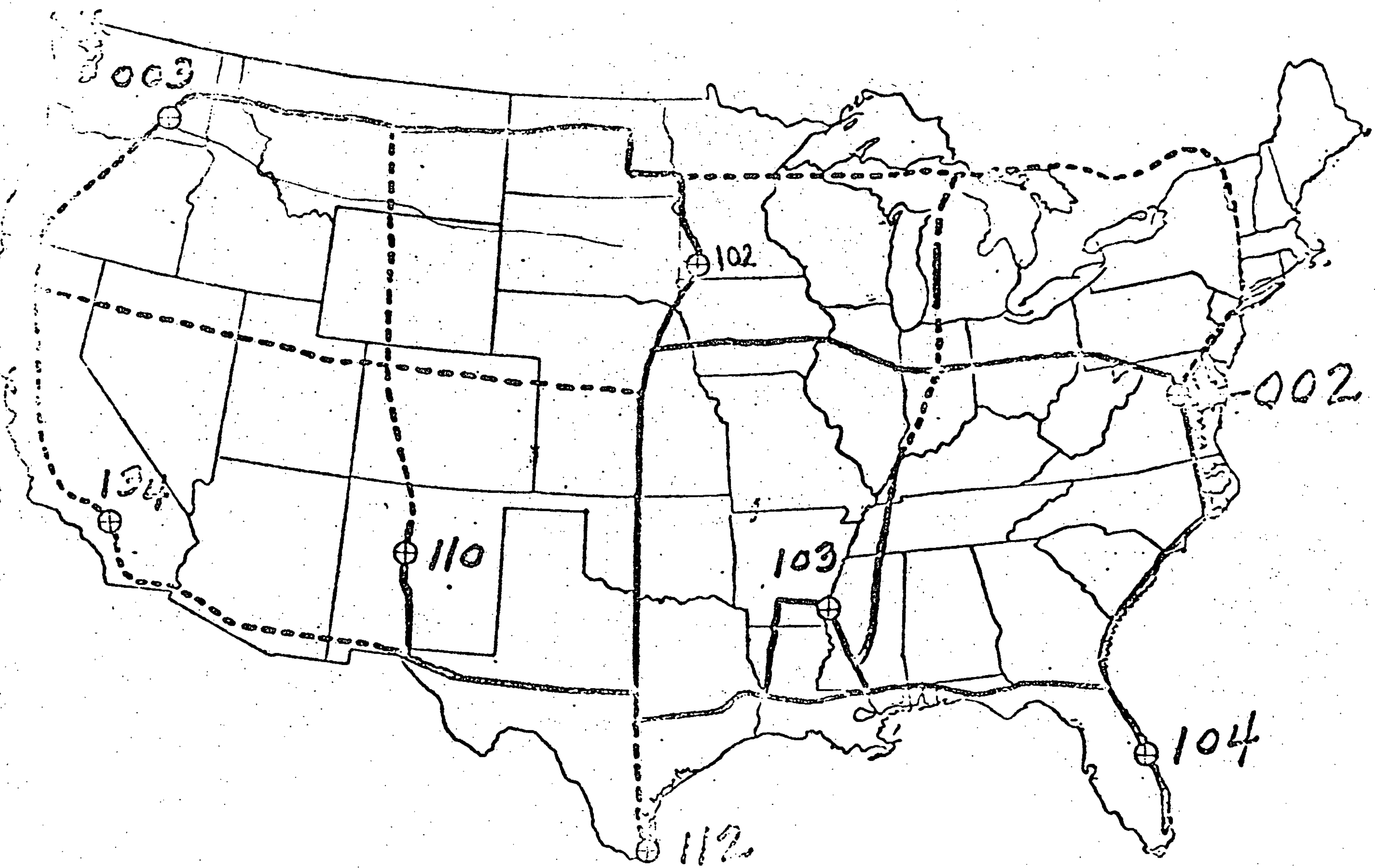


Figure 1. General Location of Geodetic Triangulation Chain from Moses Lake (003) to Chandler (102).

The details of Moses Lake - Chandler triangulation chain are as follows:

Number of stations		191
Number of bases	Taped	27
	Geodimeter	2
Laplace stations		13
Observed directions		919
Distance between two stations	Minimum	273 m
	Maximum	190 km
Total length of the chain		1858 km.

It is assumed that the necessary reductions have been applied to the observed data, and the weight function P is "a priori" known to be a sufficient good accuracy.

Super-transcontinental traverse (STT) runs across the western-half and the eastern-half of the U.S.A. (Figure 2). Its specifications, configuration, reduction of data and instrumentation are dealt with by Meade [1967; 1969a; 1969b].

Continental satellite net (CSN) is, in general, planned in such a way so that the stations are around 1200 km apart and that these stations are evenly distributed over the entire area. CSN-stations are either the stations of first-order triangulation net or these are connected to them. Its specification and configuration are dealt with in [Deker, 1967; Mueller, 1964; Pellinen, 1970; Schmid, 1970]. The continental satellite net of the North American Continent comprises of twenty stations which can be anchored in the three world net stations; Thule, Greenland, Moses Lake, Washington, and Beltsville, Maryland. Furthermore, planned is a tie to a fourth world net station - Shemya (Figure 3) [Schmid, 1970].

The following representative standard errors for observed data of Moses Lake-Chandler triangulation chain has been suggested [Meade, 1970]:

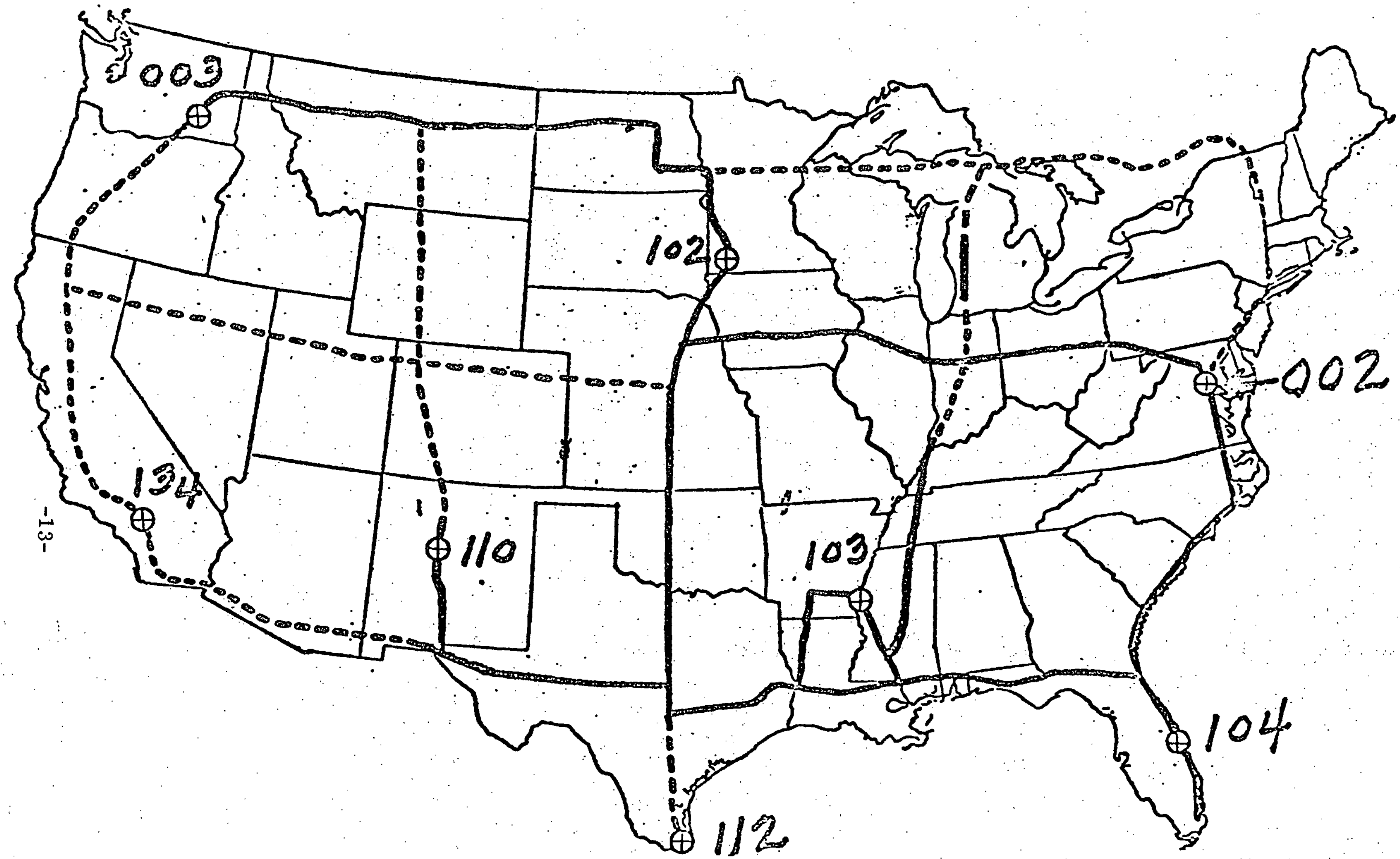


Figure 2. SUPER-TRANSCONTINENTAL TRAVERSE

— Completed July 10, 1970

⊕ Satellite Triangulation Stations

--- Proposed

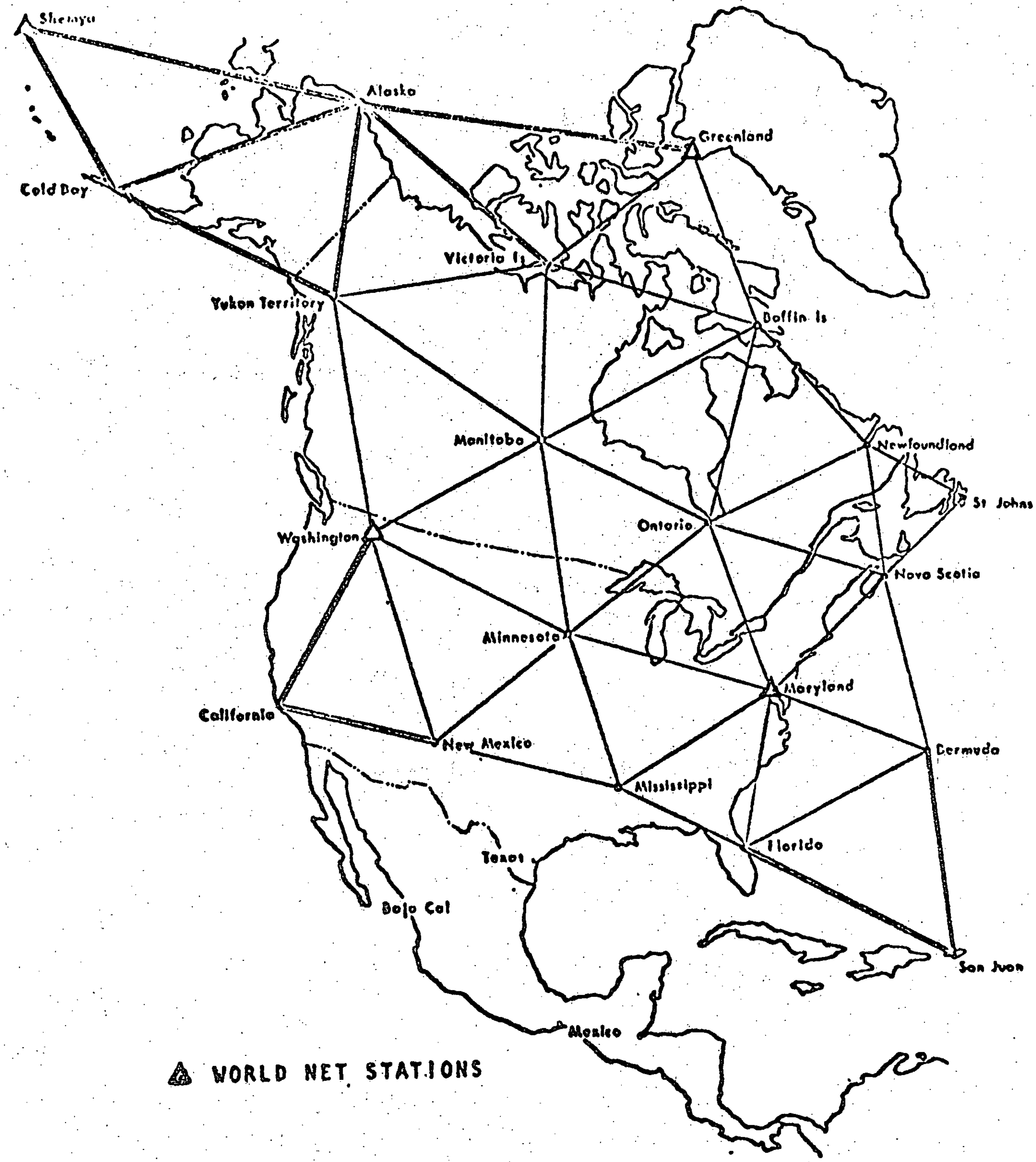


Figure 3. Continental Satellite Net of North America

Directions	$0\overset{''}{.}4$		
Azimuth	$0\overset{''}{.}8$		
Base	Taped		1 part in 500,000
	Geodimeter		1 ppm for distance > 15 km
			1.5 cm for distances up to 15 km

The mean of all section closures, which is the accuracy measure for the investigated geodetic triangulation net, is given as 1 part in 317,000 [Adams, 1930]. The standard position errors of the end stations of super-transcontinental traverse, which represent its accuracy measure, using actual data sets as given by different investigators differ too much from each other. The proportional error, which is the standard position error divided by the distance of the station from traverse-origin, is used for this investigation. The proportional errors of super-transcontinental traverse are given as follows: 1:740,000 over 318 kilometer long traverse, and 1:1,100,000 over 1270 kilometer long traverse [Foreman, 1970]; 1:670,000 over 270 kilometer long traverse [Gergen, 1970] and 1:3,000,000 over 1858 kilometer long traverse [ESSA, 1969]. The preliminary accuracy (i.e. proportional error) of continental satellite net, as obtained from the supplied data, corresponds to 1:385,000 for Chandler station. Because of this wide range in preliminary accuracy measures of these two super-control nets, investigations using the following accuracies (station constraints), are made: 1:300,000; 1:400,000; 1:500,000; 1:600,000; 1:700,000; 1:1 M; 1:1.5 M; 1:3 M. The use of these accuracy measures, which are within the limits of preliminary accuracies of the two super-control nets, will determine a limit on the accuracy requirement of the super-control net, which would be necessary to improve the geodetic triangulation net.

2.23 Computations and Results

During the earlier period of this investigation considerable thought was given to the selection and use of such formulas and methods which would not only provide high accuracies, but also minimize or eliminate loss of accuracy in computations. This resulted in using Helmert-Rainsford-Sodano's Iterative Solution for Inverse Problem, which is equally applicable for short and long lines, and Conjugate Gradient Method (Cg-Method) for the adjustment of the triangulation nets, where the original observation equation coefficient Matrix (A-Matrix) is used, thus avoiding direct formation of normal equations where certain properties of the original A-Matrix are lost. To minimize the round-off errors, computations are done in double-precision with double precision storage [Müller-Merbach, 1970].

From the two basic adjustment methods, i.e., Method of Observation Equations and Method of Condition Equations, the former has been preferred for the present investigation due to reasons of simplicity and clarity. The reasoning of this preference has been dealth with in [Grossmann, 1961, p. 174; Helmert, I. Teil, 1880, p. 556; Wolf, 1968, p. 323]. Due to the large size of the triangulation net under investigation and the availability of digital computers, iterative methods were considered because (1) they are easier to program, (2) they require less storage space as the coefficient matrix of a triangulation net is very sparse, (3) they use directly the original set of equations throughout the process and hence rounding-off errors do not accumulate from one iterative cycle to another.

While searching for a suitable adjustment method, this investigator came across the Conjugate Gradient Method (Cg-Method) [Schwarz, 1968 and 1970; Wolf, 1968], which has the following distinct advantages over other iterative methods, such as Gauss-Seidl-, Jacobi-, Relaxtion- and other Gradient methods:

1. Original A-Matrix is used, thus avoiding the formation of normal equations, where certain useful characteristics of A-Matrix, such as very small coefficients may be lost.
2. Original A-Matrix, which has very few non-zero elements, is easily stored in comparatively much less computer space using an Index-Matrix.
3. No "mesh-point numbering technique" [Ashkenazi, 1967] to keep the band-width of the system a minimum is necessary. Thus stations can be added or taken out from the existing triangulation system without caring for their numbering.
4. It is a finite iterative process. Theoretically, the solution vector is obtained in a maximum of n-steps, n being the number of unknowns. Therefore, eigenvalues need not be calculated for determining the convergence. However, experimentation shows that the solution vector is not obtained in n-steps, as the orthogonality between the residue-vectors is not maintained exactly. Consequently, the residue-vector $r^{(n)}$ after n-iterations is not zero. This deviation from zero depends upon the condition of the system; the poorer the condition, the larger will be the deviation.
6. Each approximation $x^{(j)}$ to the solution vector is closer to the true solution x than the preceeding one.
7. The ability to start anew at any point in the computation using the last $x^{(j)}$ as initial value so as to minimize the effects of round-off errors.

Following mathematical model, using method of observation equations, is used:

Let L_i be the m independent observed quantities, v_i the residuals to the observed quantities (obtained from the adjustment) and x, y, z, ... the n unknown parameters to be determined. Each observation gives an observation equation,

whose general from is

$$L_i + v_i = f_i(x, y, z, \ldots), \tag{1}$$

where $i = 1, 2, 3 \ldots, m$ and f represents a linear or non-linear function. The method of least squares however demands that (1) f should be linear, i. e., a linear relationship between the observations and the unknowns and (2) the number of observations (m) should be greater than those of the unknowns (n) i. e., $m > n$. In case of a non-linear function f this is linearized by using Taylor series about such good approximate values of the unknowns $x_0, y_0, z_0, \ldots$ such that the second and higher order terms can be neglected. In this case, equation (1) can be written as

$$v_i = a_i dx + b_i dy + c_i dz + \ldots + \ell_i \tag{2}$$

where

$$x = x_0 + dx, \quad y = y_0 + dy, \quad z = z_0 + dz, \ldots$$

$$a_i = \frac{\partial f_i}{\partial x}, \quad b_i = \frac{\partial f_i}{\partial y}, \quad c_i = \frac{\partial f_i}{\partial z}, \ldots \tag{3}$$

$$\ell_i = f_i(x_0, y_0, z_0, \ldots) - L_i .$$

Observation equation (2) can be written in the matrix form as

$$v = Ax + 1 . \tag{4}$$

It will be seen later that we have preferred to use weighted constraints to the station Chandler. These "a priori" weighted constraints on the station position generate observation equations of the form

$$v_x = Fx \tag{5}$$

where F is a rectangular matrix, whose elements are either zeros or one. Thus the complete observation equation system can be written as

$$V = Bx + L \qquad (6)$$

where

$$V = \begin{bmatrix} v \\ v_x \end{bmatrix}; \quad B = \begin{bmatrix} A \\ F \end{bmatrix}; \quad L = \begin{bmatrix} l \\ O \end{bmatrix}. \qquad (7)$$

Due to angular and linear (distance) observations, the observed data in a triangulation net are of a heterogeneous or dissimilar nature.[1] This heterogeneous data have not only more than one dimension but also different "a priori" standard errors. To make this data homogeneous, i.e., dimensionless and of unit weight, it is divided by the corresponding "a priori" standard error σ. For reasons of simplicity, the mathematical model used is assumed to be uncorrelated. The resulting homogenized observation equation system can be written as

$$\tilde{V} = \tilde{B}x + \tilde{L} \qquad (8)$$

where

$$\tilde{V} = \begin{bmatrix} \tilde{v} \\ \tilde{v}_x \end{bmatrix}; \quad \tilde{B} = \begin{bmatrix} \tilde{A} \\ \tilde{F} \end{bmatrix}; \quad \tilde{L} = \begin{bmatrix} \tilde{l} \\ O \end{bmatrix} \qquad (9)$$

and

$$\begin{aligned} &\tilde{v} = v/\sigma_l; \quad \tilde{A} = A/\sigma_l; \quad \tilde{L} = l/\sigma_l \\ &\tilde{v}_x = v_x/\sigma_x; \quad \tilde{F} = F/\sigma_x \end{aligned} \qquad (10)$$

σ_l = standard error of L_l; σ_x = standard error of x_j

[1]The term "heterogeneous or dissimilar" observations is used when the methods of their measurement are diverse; thus not only angles and distances, but also distances and heights are heterogeneous observations [Wolf, 1968, p. 56]; [Schmid and Schmid, 1965a, p. 10] uses the term "hybrid systems" for "heterogeneous systems".

Equation (8) is used directly for adjustment by conjugate gradients method. A complete algorithm for obtaining solution vector and N^{-1} by Cg-Method is given later, which gives $v^T Pv$ and Q_{xx} or Q_{yy} for a particular column. Using these quantities the "a posteriori" variance of unit weight ($\hat{m}_o^2$), standard errors (m_x, m_y) of unknowns, standard positional error (m_p) and the elements θ, A, B of the error ellipse are computed [Wolf, 1968, pp. 286-292].

The geodetic triangulation net is adjusted as an independent or free net, as it is not connected with other nets. For its unambiguous determination, besides the observed data which include directions, bases (to provide the scale) and astronomical observations, i.e., longitude and azimuth (to provide orientation of the tirangulation net upon a mathematical surface, i.e., ellipsoid), one fixed station is required to serve as origin [Gotthardt, 1968, p. 167; Grossmann, 1961, p. 175]. Moses Lake station is kept as origin with its coordinates obtained from satellite triangulation results; these coordinates have been assumed to be the best known coordinates. As Moses Lake station is fixed, its corresponding x-vector is zero, i.e., corrections $d\varphi$ and $d\lambda$ are zero. For computational ease their corresponding elements of the A-matrix are substituted with zero.

Combining the free triangulation net with super-control net of zero order, i.e., continental satellite net and/or super transcontinental traverse means constraining the scale and/or orientation of the triangulation net. The effect of this combination is comparable with "tennis racket and string effect," where the rigid outer racket frame (super-control) constrains the loose strings (triangulation net). If the strings are already constrained, there would be no "visible" effect of the additional constrain from the rigid outer frame. This is also the purpose of this investigation, i.e., to evaluate whether the existing geodetic triangulation is sufficiently "constrained" or needs to be constrained by

additional super-control net. For the present investigation triangulation station Chandler, which is common on the three networks, provides constraint.

Geodetic triangulation net can be combined with the super-control net in either of the two ways:

(1) By using the actual data, i.e., by using the actual coordinates with their standard errors of Chandler as obtained from CSN and STT with the geodetic triangulation; or,

(2) By adding a weight constraint to Chandler with its coordinates from the geodetic triangulation.

For this investigation, the first way could not be used, as the super-control net coordinates of Chandler station are not compatible with those obtained from geodetic triangulation. As such, the second way has been preferred by using the actual preliminary accuracy estimates for Chandler, which are 1 part in 385,000 and 1 part in 3 million, as obtained from CSN ans STT, respectively. Further investigations are made by using hypothetical standard positional error accuracy estimates of Chandler station, which are 1:400,000; 1:500,000; 1:600,000; 1:700,000; 1:1 M; 1:1,5 M. These accuracy estimates are within the actual preliminary accuracy estimates of super-control nets. Thus, using these various accuracies of super-control net, a feeling for the accuracy limit of super-control net, which would be necessary to improve the investigated geodetic triangulation, can be obtained.

The Method of Conjugate Gradients (Cg-Method) is a nonstationary relaxation method,

$$Nx + u = O \tag{11}$$

in n-iterative steps, where N is symmetric and positive definite. Then the system (11) - known in geodesy as the Normal Equations - has a unique solution. However, it is not necessary to have normal

equations, as Cg-Method can be easily modified for directly using the observation equations without explicit formation of normal equations. A complete mathematical derivation of Cg-Method with its program is given by Saxena [1972a; 1972b].

A complete algorithm of Cg-Method for obtaining the solution vector (x) and for obtaining N^{-1} using directly the homogenized observation equations can be summarized in the following systematic way:

A. <u>For obtaining the solution vector (x)</u>

<u>Given</u>: Homogenized Observation Equation: $Ax + 1 = v$

<u>Select:</u> Initial Trial Vector $x^{(0)} = 0$

<u>Compute:</u>

(1) $v^{(0)} = Ax^{(0)} + 1$

Relaxation steps $j = 1, 2, \ldots n$

(2) $r^{(j-1)} = A^T v^{(j)}$

(3) $\epsilon_{j-1} = \dfrac{(Ar^{(j-1)})^T (Ah^{(j-1)})}{(Ah^{(j-1)})^T (Ah^{(j-1)})}$ (for $j \geq 2$)

(4) $h^{(j)} = \begin{cases} -r^{(0)} & \text{(for } j = 1) \\ -r^{(j-1)} + \epsilon_{j-1} h^{(j-1)} & \text{(for } j \geq 2) \end{cases}$

(5) $\lambda_j = -\dfrac{r^{(j-1)T} h^{(j)}}{(Ah^{(j)})^T (Ah^{(j)})}$

(6) $x^{(j)} = x^{(j-1)} + \lambda_j h^{(j)}$

(7) $v^{(j)} = Ax^{(j)} + 1$

(7a) $v^{(j)}_{check} = v^{(j-1)} + \lambda_j Ah^{(j)}$

<u>Tests:</u>

(8) Orthogonality Test:

$$r^{(j)T} h^{(j)} = 0$$

$$r^{(j-1)T} r^{(j)} = 0$$

$$\cos \theta_1 = \frac{r^{(j-1)T} r^{(j)}}{\|r^{(j-1)}\| \cdot \|r^{(j)}\|} = 0$$

$$\cos \theta_2 = \frac{r^{(j)T} h^{(j)}}{\|r^{(j)}\| \cdot \|h^{(j)}\|} = 0$$

(9) $v^{(j)} = v^{(j)}_{check}$

Termination of Iterations.

Based upon the theory of Cg-Method and the geodetic requirements, iterations should be terminated as soon as any of the following conditions are fulfilled:

(a) if the improvement in the solution vector between two consecutive iterations is negligibily small, i. e., $|x^{(j)} - x^{(j-1)}| = 1.0 \cdot 10^{-4}$ seconds (i. e. $1.0 \cdot 10^{-4}$ second in φ or $\lambda \hat{=} 3.0$ mm),

(b) if $r^{(j)T} r^{(j)} = 0$,

(c) if $(Ah^{(j)})^T (Ah^{(j)}) = 0$;

(d) if the given number of iterations is reached;

(e) if the round-off error (RFE) during iterations exceeds a certain accuracy limit, which is given by the vector difference $|r_{true} - r_{comp}|^2$, where $r_{true} = A^T A x_j + A^T \ell = A^T v^{(j)}$ and $r_{comp} = A^T v^{(j)}_{check}$.

$$RFE = |A^T (v^{(j)} - v^{(j)}_{check})|^2$$

The iterations should be terminated if $r^{(j)T} r^{(j)} \leq 3.$ RFE.

B. For Obtaining N^{-1} - Inverse of Normal Equations

Given: Homogenized observation equation coefficient matrix A.

Select: Initial trial vector $q_k^{(o)} = O$; where q_k is the k-th column vector of $Q \ (= N^{-1})$

Compute:

(1) $r^{(o)} = -e_k$; where e_k is the k-th column vector of the unit matrix E.

(2) ϵ_{j-1}, $h^{(j)}$, λ_j are to be computed according to equations given in (A) above.

(3) $q_k^{(j)} = q_k^{(j-1)} + \lambda_j h^{(j)}$

(4) $r^{(j)} = r^{(j-1)} + \lambda_j A^T(Ah^{(j)})$

Test and Termination of Iterations: Same as in (A) above.

The algorithm of (A) is programmed as a SUBROUTINE SOLN and and (B) as a SUBROUTINE QSOLN. Both subroutines can be used for any feasible size of data, which can be accommodated on the available computer, after changing KM, which is the PATSUM Basic Block Size for RTR.

The main program used together with these subroutines has dimension statements and a data card for Number of Unknowns (NU), Number of Equations (NE) and Number of Columns of Index Matrix (NI), which can be changed if there is need for it.

The program is universal in the sense that it can be used for varying data without much change and that "mesh-point numbering technique" is not required. Therefore, stations can be added or taken out from the triangulation system without worrying about the band-width and size of blocks. These programs have been tested on systems from as small as 2 unknowns, 3 equations up to as large as 804 unknowns, 1397 equations.

Although the Cg-Method theoretically gives the solution vector at n-iterative steps (n = number of unknowns), investigations show that the solution vector is not achieved in n-iterations due to round-off errors, ill-conditioning of the system, disturbances of the orthogonality and of the conjugancy relations [Beckman, 1960, pp. 69; Hestenes and Stiefel, 1952, pp. 411]. The present investigation, using the actual data set,

shows that the number of iterations required to obtain the solution vector by Cg-Method using directly the A-matrix without explicitly forming the N-matrix depends upon two factors: (1) condition of the system, and (2) accuracy of the solution vector required.

Using the geodetic triangulation data (573 unknowns, 963 equations), the program went up to 5778 iterations without giving any 7 decimal accurate solution vector, while 4 decimal accurate solution vector was obtained after 1161 iterations, i.e., 2.1 times number of unknowns (Table 1).

Each column vector q_k of N^{-1} is generally computed in less than 1.2 n-iterations (Table 1).

Table 1.

Experiment Number*	Number of Unknowns	Number of Equations	Solution Vector Iterations	Solution Vector Time** m	Solution Vector Time** sec	Covariance Vector for Column 8 Iterations	Covariance Vector for Column 8 Time** m	Covariance Vector for Column 8 Time** sec
1	573	963	1161	9	37.13	640	3	45.96
2	573	965	1177	9	23.27	657	3	31.91
3	573	965	1175	5	45.97+	659	2	12.59+
4	573	965	1176	9	22.32	682	3	45.64
5	573	965	1164	5	53.44+	674	2	1.77+
6	573	965	1162	5	41.16+	675	2	0.00+
7	573	965	1166	9	09.46	631	3	20.03
8	573	965	1159	9	24.29	648	3	19.29
9	573	965	1169	9	29.41	608	3	11.51

*Refer to Table 2.

**Time is the Execution time on H-Compiler, Option = 2 (IBM 360/75) except those marked with a plus (+) sign, which is the Execution time on H-Compiler, Option = 0 (IBM 370/165).

The results of the investigation are given in Table 2 and 3, wherein the improvement of the particular geodetic triangulation by super-control net is visible only when its accuracy is at least 1 part in 500,000.

Table 2.

Experiment Number	Accuracy 1 in	$\hat{m}_o$	WYOLA (95)				Remarks
			Q_{xx}	Q_{yy}	m_x^2	m_y^2	
1		2.42	6.0	0.5	35.2	2.9	Free Net
2	300,000	2.41	6.7	0.5	38.9	2.9	
3	400,000	2.41	5.9	0.5	34.3	2.9	
4	500,000	2.41	4.1	0.5	23.8	2.9	
5	600,000	2.41	4.1	0.5	23.8	2.9	
6	700,000	2.41	4.1	0.5	23.8	2.9	
7	1,000,000	2.41	3.7	0.5	21.5	2.9	
8	1,500,000	2.41	3.2	0.5	18.6	2.9	
9	3,000,000	2.41	2.1	0.5	12.2	2.9	

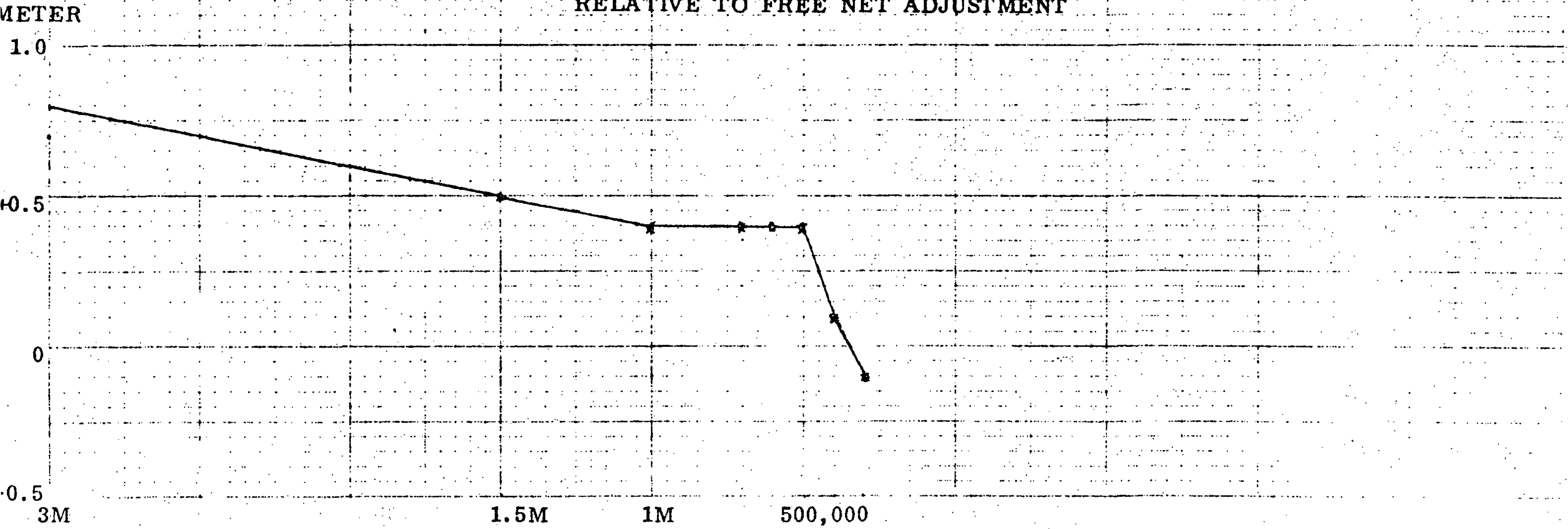
Q_{xx}, Q_{yy} and m_x^2, m_y^2 are given in 10^{-4} seconds^2.

Table 3.

Experiment Number	Accuracy 1 in	WYOLA (95)				
					Positional Improvement Relative to Experiment 1	
		m_x	m_y	m_p	Meters	%
1	Free Net	1.83	0.37	1.9		
2	300,000	1.93	0.37	2.0	-0.1	- 5
3	400,000	1.81	0.37	1.8	0.1	5
4	500,000	1.51	0.37	1.5	0.4	21
5	600,000	1.51	0.37	1.5	0.4	21
6	700,000	1.51	0.37	1.5	0.4	21
7	1,000,000	1.43	0.37	1.5	0.4	21
8	1,500,000	1.33	0.37	1.4	0.5	26
9	3,000,000	1.08	0.37	1.1	0.8	42

Standard Errors of Unknowns (m_x, m_y) and Standard Positional Error (m_p) are given in meters.

Figure 4.
POSITIONAL IMPROVEMENT
OF WYOLA
RELATIVE TO FREE NET ADJUSTMENT



Worth mentioning is that the longitude terms, which are Q_{yy} and m_y^2 in Table 2 remain practically uneffected. This could be explained by the fact that station Wyola is very close to Laplace stations, which control the azimuth error accumulation and thus control the longitude terms.

It is interesting to note from Table 3 and Figure 4 of the investigated geodetic triangulation chain lies between 1:300,000 and 1:400,000, which is quite in agreement with its presumed accuracy of 1:317,000.

2.24 Summary and Conclusions

The super-control net, i.e., continental satellite net or super-transcontinental traverse, can provide a useful constraint to the investigated geodetic triangulation net, and thus can improve it only when the accuracy of super-control net is at least 1 part in 500,000; in this case, this corresponds to ±3.7 m standard position error for the station Chandler.

The preliminary accuracy of super-transcontinental traverse is already better than this limiting accuracy of 1 part in 500,000. The preliminary accuracy of continental satellite net is, however, lower than the limiting accuracy of 1:500,000; the preliminary standard position error for Chandler as obtained from continental satellite net corresponds to ±4.8 m, i.e., 1:385,000. The future will show whether the limiting accuracy could be achieved by continental satellite net, especially because numerous spatial triangulations of CSN have produced accuracies within the range of 1 part of 400,000 and 1 part in 700,000 [Schmid, 1965, p.22].

Schmid [1970, pp. 23-24] indicates that continental satellite net will fall short on an optimum solution with respect to both its coverage and its accuracy. The three-dimensional positions of CSN-stations will

probably be determined to no better than ±4 meters in all components, which does not seem to be good enough at least for this particular investigation.

It might be useful to have a "block constrain" instead of "chain constrain", that is, to use four well separated satellite stations, namely 003, 102, 112 and 134 (Figure 1).

Super-transcontinental traverse can provide a better constraint, if more than two of its stations are common to the stations of geodetic triangulation net. Also, a "block constrain", as explained above, might be more useful instead of a "chain constrain".

The development tendencies of instrumentation indicates that the future super-control nets will use VLBI (Very Long Baseline Interferometry) and Laser ranging systems.

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2.3 Geodetic Satellite Observations in North America (Solution NA9)

The coordinates of several tracking stations tied to the NAD datum were computed through available observations to the GEOS-I satellite. Up to date the NA6 adjustment [Mueller, Reilly and Schwarz, 1969] and NA8 adjustment [Mueller, and Reilly, 1971] had been published. The latter solution was performed using height constraints deduced from the SAO69 geoid [Gaposchkin and Lambeck, 1970].

Recently a new detailed geoidal map with claimed accuracies of ±2 m, (on land), based on gravimetric and satellite data, was presented [Vincent, Strange and Marsh, 1971]. With the new geoid, and the orthometric heights given in [NASA, 1971] more reliable height constraints were calculated as follows:

From the initial values of the shifts SAO-NA8 (computed using the published shifts SAO-NAD and NAD-NA8 in [Mueller and Reilly, 1971]) and by an iterative process self-explained in Figure 1, the initial NA8 rectangular coordinates were shifted to the SAO origin and the geodetic coordinates computed. The ellipsoidal heights then were constrained using the undulations from [Vincent, Strange and Marsh, 1971]. With the original φ and λ and this new height a new set of rectangular coordinates was obtained. Following this procedure iteratively, several shifts of this kind to the "geocenter" were performed until the sum of the undulation differences was very small. Through this process "best" shift to the geocenter was obtained. This shift was also used to compute the preliminary coordinates to obtain the reduced normal equations for the MOTS and PC-1000 optical data in the solution MPS7 ([Mueller and Whiting, 1972] and Section 2.4).

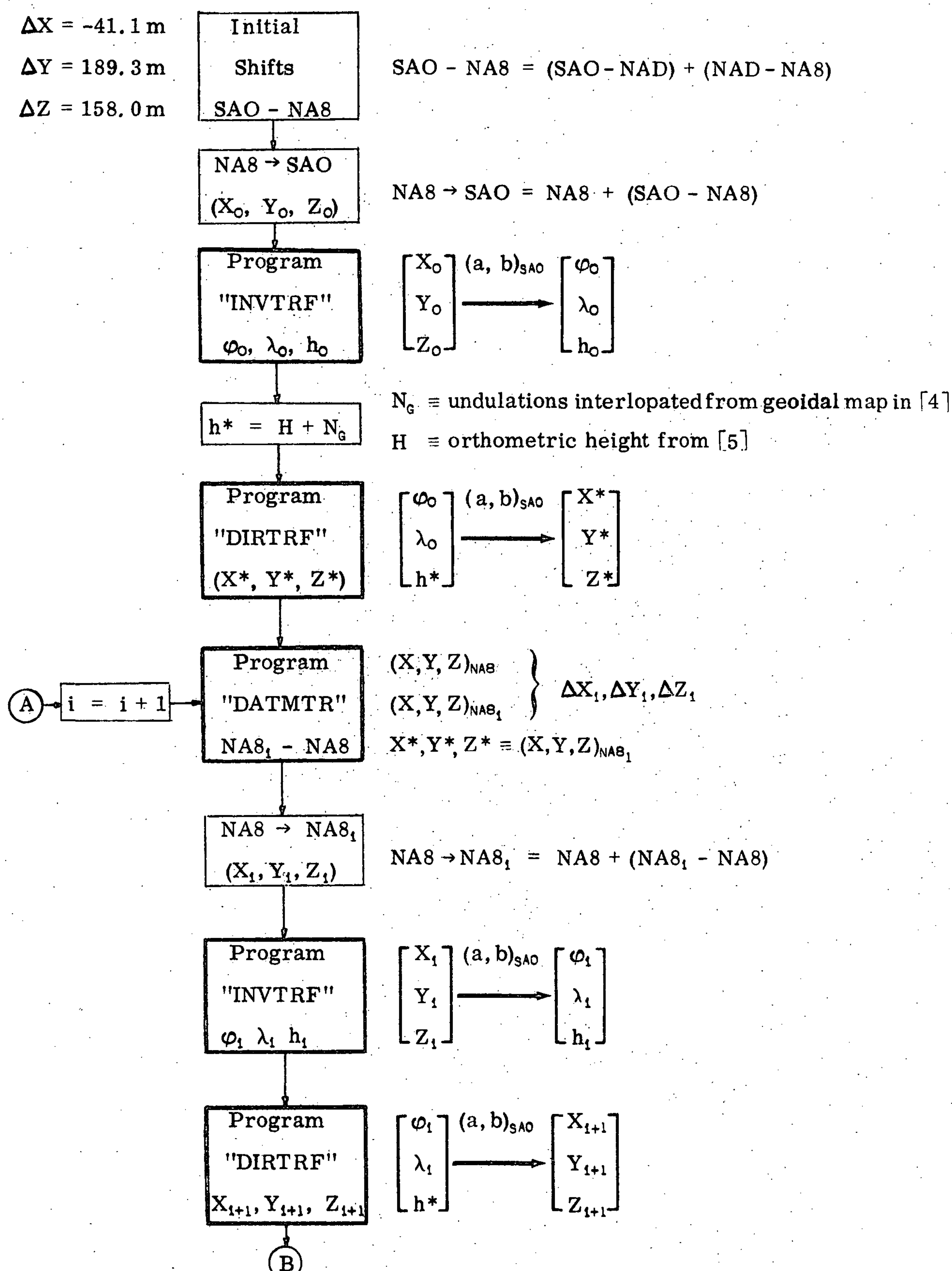
At all stations, a weighted height constraint was imposed, after shifting (with the above obtained values) to the final "geocentric (GC)" coordinates. Also, as in the NA8 adjustment, a distance constraint was imposed between stations 3861 and 7043. Due to a recent correction in their coordinates a difference of 3 m from the previously used value was taken into consideration [Meade, 1972].

Finally, unlike in the NA8 solution, "inner adjustment constraints" were also imposed in order to define the origin of the system in its most favorable position from the error propagation point of view [Blaha, 1971].

The coordinates of the NA9 solution are presented in Table 1 with corresponding standard deviations. The coordinates transformed to the NAD datum are in Table 2.

Table 3 shows the constrained heights at each station and the final undulations compared with those published by Vincent et al. In column ΔN and in parenthesis, the differences published previously in [Vincent, Strange and Marsh, 1971] are shown. It can be seen from Table 3 that only two stations show substantial disagreements (3903 Herdon, Virginia and 3407 Trinidad). It seems clear that the orthometric height as given in [NASA, 1971] for the station 3903 has a gross error. Appropriately, the NASA Directory of Tracking Stations points out in the description of the referred station: "coordinates unverified, survey details are lacking." The discrepancy with respect to station 3407 may be due to the fact that it is situated in the Caribbean, where large geoidal gradients are present.

Table 4 shows the transformation parameters between the different systems. Listed on the first page are the 3 parameter-transformation solutions (only shifts considered), and the general 7 parameter solution. In this latter transformation the rotations were first computed through direction cosines independent of translations and scale factor (see Section 2.5). These rotation parameters constrained with their variances were used in the final solution shown pp. 2-5 of Table 4 with the resulting variance-covariance matrix and the correlation coefficient matrix for each transformation. In the variance-covariance matrix the angular units are in radians.



ΔX = -41.1 m
ΔY = 189.3 m
ΔZ = 158.0 m
Initial Shifts SAO - NA8
SAO - NA8 = (SAO - NAD) + (NAD - NA8)
NA8 → SAO (Xo, Yo, Zo)
NA8 → SAO = NA8 + (SAO - NA8)
Program "INVTRF" φo, λo, ho
(a, b)SAO
NG ≡ undulations interlopated from geoidal map in [4]
H ≡ orthometric height from [5]
h* = H + NG
Program "DIRTRF" (X*, Y*, Z*)
(a, b)SAO
A
i = i + 1
Program "DATMTR" NA8i - NA8
(X, Y, Z)NA8
(X, Y, Z)NA8i
ΔXi, ΔYi, ΔZi
X*, Y*, Z* ≡ (X, Y, Z)NA8i
NA8 → NA8i (Xi, Yi, Zi)
NA8 → NA8i = NA8 + (NA8i - NA8)
Program "INVTRF" φi λi hi
(a, b)SAO
Program "DIRTRF" Xi+1, Yi+1, Zi+1
(a, b)SAO
B

Figure 1

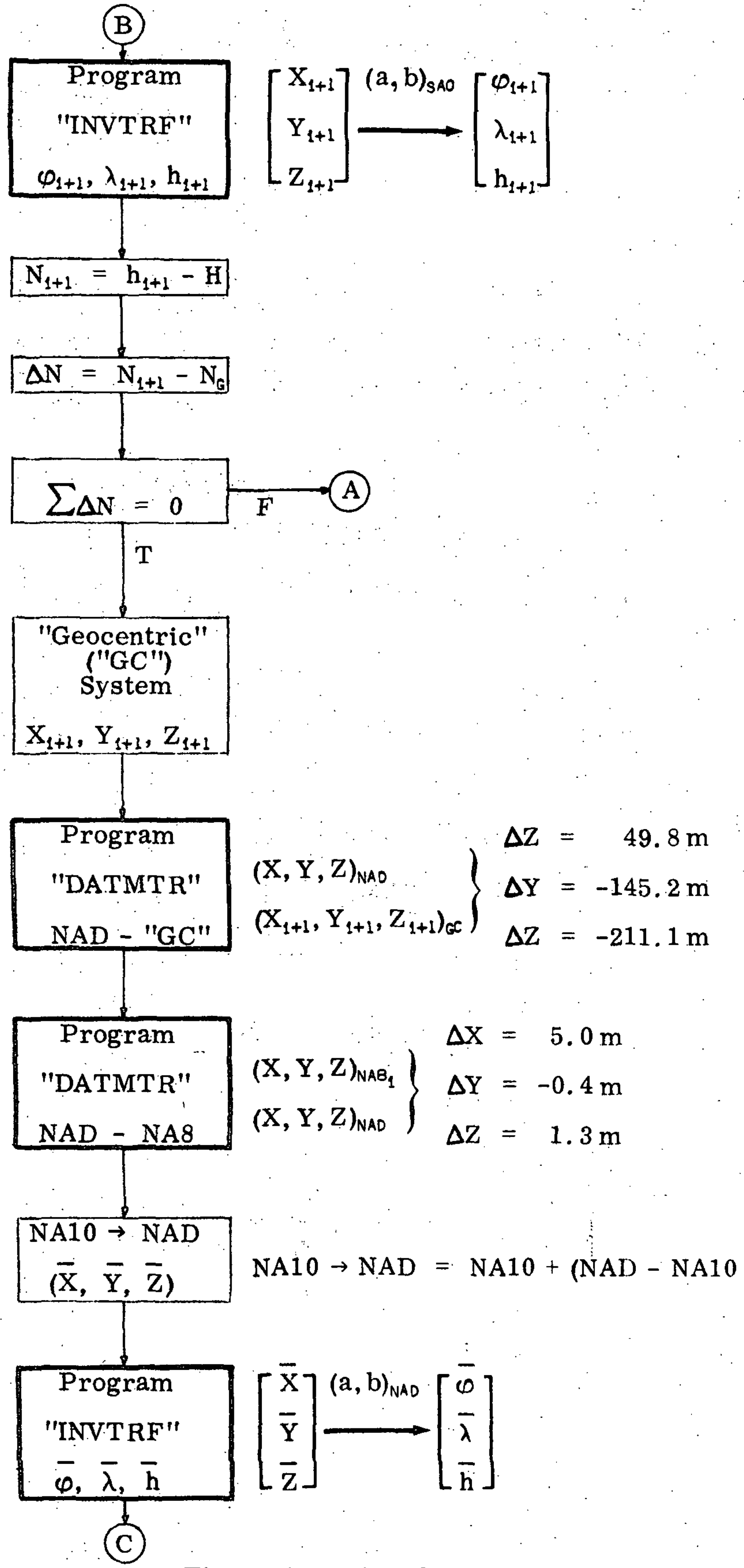


Figure 1 continued

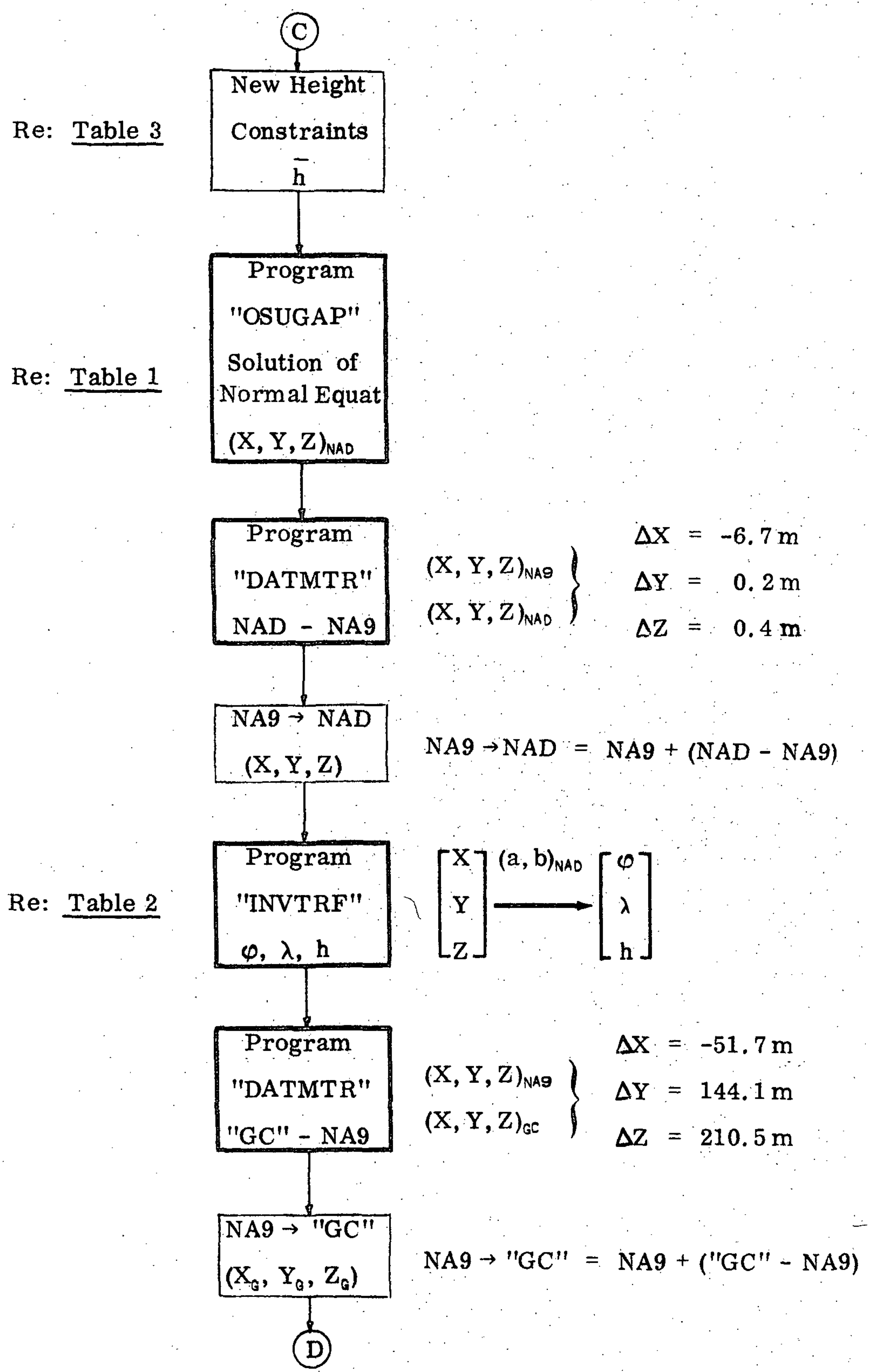


Figure 1 continued

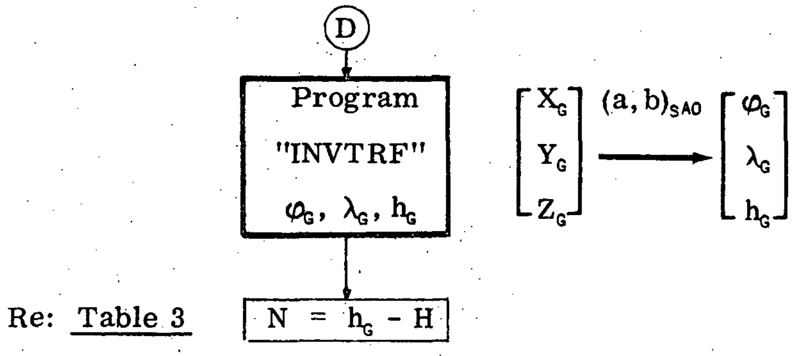


Figure 1

Table 1

Coordinates of the North American GEOS-I Tracking Stations from the NA-9 Geometric Adjustment

Station	Name		NA-9	σ
1021	Blossom Point,	X	1,118,058.5	3.5
	Maryland	Y	-4,876,470.6	3.1
	MOTS 40	Z	3,942,797.3	3.1
1022	Ft. Myers,	X	807,891.2	2.6
	Florida	Y	-5,652,138.0	1.8
	SECOR	Z	2,833,333.7	1.9
1030	Mojave,	X	-2,357,203.5	6.6
	California	Y	-4,646,475.7	3.6
	MOTS 40	Z	3,668,126.3	2.9
1032	St. Johns,	X	2,602,615.5	50.2
	Newfoundland	Y	-3,419,495.8	60.3
	MOTS 40	Z	4,697,433.4	17.8
1033	College,	X	-2,299,222.5	11.7
	Alaska	Y	-1,445,847.7	33.0
	MOTS 40	Z	5,751,612.2	9.8
1034	E. Grand Fork,	X	-521,668.5	3.3
	Minnesota	Y	-4,242,203.9	4.1
	MOTS 40	Z	4,718,538.8	3.7
1042	Rosman,	X	647,535.9	2.9
	N.C.	Y	-5,178,081.5	2.4
	MOTS 40	Z	3,656,535.6	2.7
3106	Antigua Island	X	2,881,871.4	6.6
	PC-1000	Y	-5,372,319.1	3.2
		Z	1,868,378.9	5.0

Station	Name		NA-9	σ
3334	Greenville,	X	-84,952.3	4.8
	Mississippi	Y	-5,328,109.6	4.1
	PC-1000	Z	3,493,270.2	5.6
3400	Colorado Springs,	X	-1,275,159.7	9.8
	Colorado	Y	-4,798,170.9	6.0
	PC-1000	Z	3,994,031.1	5.9
3401	L.G.Hanscom Field	X	1,513,173.2	4.0
	Massachusetts	Y	-4,463,721.3	4.8
	PC-1000	Z	4,282,882.2	3.6
3402	Semmes,	X	167,298.3	3.7
	Alabama	Y	-5,482,118.2	2.4
	PC-1000	Z	3,244,869.0	3.0
3404	Swan Island	X	642,529.0	4.5
	PC-1000	Y	-6,054,080.7	4.1
		Z	1,895,526.1	4.9
3405	Grand Turk	X	1,919,526.3	5.3
	PC-1000	Y	-5,621,238.9	3.7
		Z	2,315,604.7	4.9
3406	Curacao	X	2,251,839.7	5.3
	PC-1000	Y	-5,817,062.0	3.5
		Z	1,327,045.3	6.7
3407	Trinidad	X	2,979,923.1	8.2
	PC-1000	Y	-5,513,690.2	4.6
		Z	1,180,998.1	8.9

All coordinates and standard deviations in meters.

Table 1 continued

Station	Name		NA-9	σ
3648	Hunter AFB,	X	832,601.0	3.4
	Georgia	Y	-5,349,695.7	2.0
	PC-1000	Z	3,360,417.6	1.9
3657	Aberdeen,	X	1,186,824.4	3.6
	Maryland	Y	-4,785,338.5	3.6
	PC-1000	Z	4,032,711.9	3.2
3861	Homestead AFB,	X	961,808.8	3.2
	Florida	Y	-5,679,305.3	1.8
	PC-1000	Z	2,729,717.8	2.1
3902	Cheyenne,	X	-1,234,650.6	9.5
	Wyoming	Y	-4,651,378.2	7.5
	PC-1000	Z	4,174,579.9	6.6
3903	Herndon,	X	1,089,019.6	10.9
	Virginia	Y	-4,843,098.8	13.8
	PC-1000	Z	3,991,565.8	9.6
5001	Herndon,	X	1,088,888.9	7.1
	Virginia	Y	-4,843,086.1	3.4
	SECOR	Z	3,991,668.3	4.4
5333	Stoneville	X	-84,970.6	4.8
	Mississippi	Y	-5,328,107.2	4.1
	SECOR	Z	3,493,279.1	5.6
5649	Hunter AFB,	X	832,518.4	3.4
	Georgia	Y	-5,349,735.2	2.0
	SECOR	Z	3,360,372.7	1.9
5861	Homestead AFB,	X	963,509.5	3.2
	Florida	Y	-5,679,870.0	1.8
	SECOR	Z	2,727,956.8	2.1

Station	Name		NA-9	σ
7036	Edinburg,	X	-828,452.3	3.8
	Texas	Y	-5,657,613.0	2.3
	MOTS 40	Z	2,816,643.7	2.8
7037	Columbia,	X	-191,252.5	2.9
	Missouri	Y	-4,967,435.9	2.6
	MOTS 40	Z	3,983,080.7	2.8
7039	Bermuda	X	2,308,250.0	5.7
	Island	Y	-4,873,750.4	3.9
	MOTS 40	Z	3,394,386.7	3.7
7040	San Juan,	X	2,465,089.0	5.7
	P.R.	Y	-5,535,083.1	3.3
	MOTS 40	Z	1,985,352.6	4.2
7043	GSFC, Greenbelt,	X	1,130,747.0	3.5
	Maryland	Y	-4,831,473.8	2.7
	PTH-100	Z	3,993,963.5	2.3
7045	Denver,	X	-1,240,426.6	4.6
	Colorado	Y	-4,760,380.9	3.4
	MOTS 40	Z	4,048,802.8	3.3
7072	Jupiter,	X	976,302.7	3.1
	Florida	Y	-5,601,547.9	2.3
	MOTS 40	Z	2,880,074.9	2.5
7075	Sudbury,	X	692,653.0	4.2
	Ontario	Y	-4,347,221.9	4.7
	MOTS 40	Z	4,600,298.9	4.2
7076	Jamaica,	X	1,384,194.0	4.8
	B.W.I.	Y	-5,905,810.2	3.1
	MOTS 40	Z	1,966,381.9	5.3

General Information:

No. of ground stations	34
No. of spatial chord equations	1
No. of degrees of freedom	5228
Quadratic sum of the residuals ($V'PV$)	5039
Standard deviation of unit weight	1.0

Table 2

NAD Coordinates of the North American GEOS-I Tracking Stations

Station	Name		NAD	σ
1021	Blossom Point,	φ	38° 25′ 49.″81	0.11
	Maryland	λ	282 54 47.80	0.15
	MOTS 40	h	6.4 m	2.3m
1022	Ft. Myers,	φ	26 32 52.07	0.06
	Florida	λ	278 8 3.91	0.09
	SECOR	h	24.5	1.7
1030	Mojave,	φ	35 19 47.92	0.10
	California	λ	243 6 2.81	0.28
	MOTS 40	h	900.8	2.4
1032	St. Johns,	φ	47 44 27.90	0.83
	Newfoundland	λ	307 16 30.26	3.65
	MOTS 40	h	99.0	4.9
1033	College,	φ	64 52 19.60	0.47
	Alaska	λ	212 9 47.92	2.43
	MOTS 40	h	148.6	9.5
1034	E. Grand Fork,	φ	48 1 21.19	0.16
	Minnesota	λ	262 59 21.71	0.17
	MOTS 40	h	255.2	2.2
1042	Rosman,	φ	35 12 7.03	0.09
	N.C.	λ	277 7 40.55	0.12
	MOTS 40	h	914.2	2.1
3106	Antigua Island	φ	17 8 52.81	0.17
	PC-1000	λ	298 12 37.29	0.23
		h	5.4	2.4

Station	Name		NAD	σ
3334	Greenville,	φ	33° 25′ 31.″40	0.21
	Mississippi	λ	269 5 11.24	0.18
	PC-1000	h	43.2	2.0
3400	Colorado Springs,	φ	39 0 22.22	0.24
	Colorado	λ	255 7 1.29	0.40
	PC-1000	h	2190.0	4.6
3401	L.G.Hanscom Field	φ	42 27 18.01	0.14
	Massachusetts	λ	288 43 34.46	0.20
	PC-1000	h	82.2	3.4
3402	Semmes,	φ	30 46 49.58	0.10
	Alabama	λ	271 44 52.33	0.14
	PC-1000	h	79.8	2.3
3404	Swan Island	φ	17 24 17.13	0.16
	PC-1000	λ	276 3 29.28	0.15
		h	56.1	4.1
3405	Grand Turk	φ	21 25 46.80	0.16
	PC-1000	λ	288 51 13.68	0.19
		h	-5.4	3.6
3406	Curacao	φ	12 5 23.36	0.22
	PC-1000	λ	291 9 42.55	0.18
		h	38.8	3.4
3407	Trinidad	φ	10 44 32.60	0.29
	PC-1000	λ	298 23 21.59	0.28
		h	268.5	3.9

Note: The above coordinates were arrived at by applying the shifts $\Delta X = -6.7m$, $\Delta Y = 0.2m$ and $\Delta Z = 0.4m$ to the NA-9 coordinates and then converting these values to ellipsoidal coordinates on the ellipsoid $a = 6378206.4$, $b = 6356583.8$.

Table 2 continued

Station	Name		NAD			σ
3648	Hunter AFB,	φ	32°	0′	5.″88	0.08
	Georgia	λ	278	50	46.26	0.13
	PC-1000	h			23.7m	1.4m
3657	Aberdeen,	φ	39	28	19.25	0.12
	Maryland	λ	283	55	44.15	0.16
	PC-1000	h			6.2	2.5
3861	Homestead AFB,	φ	25	30	25.08	0.07
	Florida	λ	279	36	43.00	0.12
	PC-1000	h			15.2	1.7
3902	Cheyenne,	φ	41	7	58.01	0.29
	Wyoming	λ	255	8	3.32	0.40
	PC-1000	h			1883.0	4.9
3903	Herndon,	φ	38	59	34.36	0.34
	Virginia	λ	282	40	21.58	0.45
	PC-1000	h			95.1	13.4
5001	Herndon,	φ	38	59	37.78	0.16
	Virginia	λ	282	40	16.40	0.29
	SECOR	h			127.7	2.7
5333	Stoneville	φ	33	25	31.68	0.21
	Mississippi	λ	269	5	10.53	0.18
	SECOR	h			46.4	2.0
5649	Hunter AFB,	φ	32	0	4.19	0.08
	Georgia	λ	278	50	42.92	0.13
	SECOR	h			22.3	1.4
5861	Homestead AFB,	φ	25	29	21.66	0.07
	Florida	λ	279	37	39.66	0.12
	SECOR	h			16.2	1.7

Station	Name		NAD			σ
7036	Edinburg,	φ	26°	22′	45.″46	0.09
	Texas	λ	261	40	9.19	0.13
	MOTS 40	h			75.2	2.3
7037	Columbia,	φ	38	53	35.84	0.10
	Missouri	λ	267	47	42.13	0.12
	MOTS 40	h			276.5	2.1
7039	Bermuda	φ	32	21	48.92	0.13
	Island	λ	295	20	33.20	0.24
	MOTS 40	h			23.1	2.6
7040	San Juan,	φ	18	15	26.46	0.14
	P.R.	λ	294	0	21.85	0.20
	MOTS 40	h			58.4	3.0
7043	GSFC, Greenbelt,	φ	39	1	15.58	0.08
	Maryland	λ	283	10	19.91	0.15
	PTH-100	h			50.9	2.4
7045	Denver,	φ	39	38	48.06	0.12
	Colorado	λ	255	23	41.80	0.20
	MOTS 40	h			1793.0	2.4
7072	Jupiter,	φ	27	1	13.29	0.08
	Florida	λ	279	53	12.39	0.12
	MOTS 40	h			26.3	2.1
7075	Sudbury,	φ	46	27	21.11	0.18
	Ontario	λ	279	3	10.33	0.21
	MOTS 40	h			278.0	2.6
7076	Jamaica,	φ	18	4	32.60	0.18
	B.W.I.	λ	283	11	26.57	0.16
	MOTS 40	h			474.8	2.7

Table 3

Height Constraints and Undulations
(all units in meters)

Number	Station	Constraints $\bar{h}$	Constraints σ	N "GC"*	N [Vincent et al.]	ΔN
1021	Blossom Pt., Md.	9	3	- 27	-26	- 1 (- 7)
1022	Fort Myers, Florida	23	3	- 16	-18	2 (1)
1030	Goldstone, Calif.	898	3	- 23	-27	4 (8)
1032	St. John's, Nswf.	102	5	12	13	- 1
1033	Fairbanks, Alaska	165	10	16		
1034	E. Grand Forks, Minn.	256	3	- 13	-18	5 (11)
1042	Rosman, N.C.	916	3	- 23	-22	- 1 (- 3)
3106	Antigua, W.I.	8	3	- 45	-40	- 5 (- 2)
3334	Stoneville, Mississippi	45	3	- 20	-19	- 1
3400	Colorado Springs, Col.	2184	5	- 4	-10	6
3401	Bedford, Mass.	89	5	- 27	-21	- 6
3402	Semmes, Alabama	84	3	- 21	-18	- 3 (-12)
3404	Swan Island	79	7	- 32		
3405	Grand Turk, B.I.	0	5	- 51	-47	- 4 (-27)
3406	Curacao, N. Antilles	44	5	- 30	-26	- 4
3407	Trinidad, Tobago	285	5	- 50	-34	-16
3648	Hunter AFB, Georgia	19	3	- 19	-24	5 (- 5)
3657	Aberdeen, Maryland	7	3	- 27	-26	- 1 (- 4)
3861	Homestead, Florida	16	3	- 22	-22	0
3902	Cheyenne, Wyoming	1882	5	- 8	-10	2
3903	Herndon, Virginia	132	3	-100	-26	-74
5001	Herndon, Virginia	132	3	- 27	-26	- 1
5333	Stoneville, Mississippi	45	3	- 17	-19	2
5649	Hunter AFB, Georgia	23	5	- 23	-23	0
5861	Homestead, Florida	22	3	- 27	-22	- 5 (- 5)
7036	Edinburg, Texas	72	3	- 8	-11	3
7037	Columbia, Missouri	270	3	- 17	-24	7 (10)
7039	Bermuda	26	3	- 38	-36	- 2 (- 2)
7040	San Juan, P.R.	57	5	- 40	-41	1 (5)
7043	Greenbelt, Maryland	56	3	- 30	-26	- 4 (-15)
7045	Denver, Colorado	1787	3	- 7	-13	6 (16)
7072	Jupiter, Florida	26	3	- 23	-24	1 (1)
7075	Sudburg, Canada	276	3	- 28	-31	3 (20)
7076	Kingston, Jamaica	473	3	- 20	-23	3 (20)

*The geocentric coördinates were obtained from the NA-9 by adding the following shifts: ΔX = -51.7 m, ΔY = 144.1 m, ΔZ = 210.5 m.

Table 4

Transformation Parameters

		NA9-NAD	NA9-"GC"	NA9-SAO	"GC"-NAD
No. Stations		32	34	11	32
3 param. transf.	ΔX(m)	6.7 ±1.3	51.7 ±0.7	35.3 ± 2.6	-44.8 ±1.3
	ΔY(m)	- 0.2 ±1.1	-144.1 ±0.8	-148.3 ± 2.6	144.8 ±1.1
	ΔZ(m)	- 0.4 ±1.2	-210.5 ±0.9	-175.0 ± 2.6	209.8 ±1.2
7 parameter transf.*	ΔX(m)	- 1.9 ±3.0	39.1 ±1.7	24.9 ± 9.4	-37.8 ±2.6
	ΔY(m)	-20.9 ±5.0	-144.4 ±4.0	-200.0 ±11.7	124.3 ±5.0
	ΔZ(m)	23.5 ±4.3	-202.5 ±2.8	-173.5 ±11.1	227.7 ±4.0
	θ_X (″)	- 0.80 ±0.09	- 0.37 ±0.04	- 0.66 ± 0.29	- 0.35 ±0.07
	θ_Y (″)	0.56 ±0.07	- 0.22 ±0.03	0.14 ± 0.24	0.84 ±0.06
	θ_Z (″)	- 0.25 ±0.11	- 0.22 ±0.05	0.94 ± 0.35	- 0.10 ±0.09
	$\epsilon(\times 10^{-6})$	- 4.82 ±0.89	- 0.48 ±0.72	- 6.78 ± 1.88	- 4.28 ±0.89

*Rotation parameters constrained (see Section 2.5)

Table 4 continued

ROTATION PARAMETERS CONSTRAINED

"GC"-NAD

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

ΔX METERS	ΔY METERS	ΔZ METERS	ϵ ($\times 10^{-6}$)	θ_z SECONDS	θ_y SECONDS	θ_x SECONDS
-37.78	124.34	227.66	-4.28	-0.35	0.84	-0.10

VARIANCE - COVARIANCE MATRIX

0.665D+01	0.103D+00	0.339D+01	-0.330D-06	0.675D-06	0.358D-06	-0.454D-06
0.103D+00	0.246D+02	-0.115D+02	0.409D-05	0.242D-06	0.978D-07	-0.597D-06
0.339D+01	-0.115D+02	0.161D+02	-0.289D-05	0.382D-06	0.111D-06	-0.960D-06
-0.330D-06	0.409D-05	-0.289D-05	0.797D-12	-0.831D-14	0.312D-15	0.980D-14
0.675D-06	0.242D-06	0.382D-06	-0.831D-14	0.117D-12	0.204D-13	-0.702D-13
0.358D-06	0.978D-07	0.111D-06	0.312D-15	0.204D-13	0.724D-13	-0.260D-13
-0.454D-06	-0.597D-06	-0.960D-06	0.980D-14	-0.702D-13	-0.260D-13	0.183D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.801D-02	0.328D+00	-0.143D+00	0.765D+00	0.517D+00	-0.412D+00
0.801D-02	0.100D+01	-0.580D+00	0.923D+00	0.142D+00	0.732D-01	-0.281D+00
0.328D+00	-0.580D+00	0.100D+01	-0.806D+00	0.278D+00	0.103D+00	-0.559D+00
-0.143D+00	0.923D+00	-0.806D+00	0.100D+01	-0.272D-01	0.130D-02	0.257D-01
0.765D+00	0.142D+00	0.278D+00	-0.272D-01	0.100D+01	0.221D+00	-0.479D+00
0.517D+00	0.732D-01	0.103D+00	0.130D-02	0.221D+00	0.100D+01	-0.226D+00
-0.412D+00	-0.281D+00	-0.559D+00	0.257D-01	-0.479D+00	-0.226D+00	0.100D+01

Table 4 continued

ROTATION PARAMETERS CONSTRAINED

NA9 - SAO

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

ΔX METERS	ΔY METERS	ΔZ METERS	ϵ ($\times 10^{-6}$)	θ_z SECONDS	θ_y SECONDS	θ_x SECONDS
24.89	-200.00	-173.46	-6.70	-0.66	0.14	0.94

VARIANCE - COVARIANCE MATRIX

0.887D+02	0.165D+02	0.280D+02	-0.152D-05	0.111D-04	0.701D-05	-0.515D-05
0.165D+02	0.138D+03	-0.111D+02	0.178D-04	0.394D-05	0.134D-05	-0.110D-04
0.280D+02	-0.111D+02	0.124D+03	-0.129D-04	0.381D-05	0.850D-06	-0.142D-04
-0.152D-05	0.178D-04	-0.129D-04	0.354D-11	0.116D-13	0.766D-14	-0.230D-13
0.111D-04	0.394D-05	0.381D-05	0.116D-13	0.192D-11	0.405D-12	-0.811D-12
0.701D-05	0.134D-05	0.850D-06	0.766D-14	0.405D-12	0.135D-11	-0.304D-12
-0.515D-05	-0.110D-04	-0.142D-04	-0.230D-13	-0.811D-12	-0.304D-12	0.289D-11

COEFFICIENTS OF CORRELATION

0.100D+01	0.149D+00	0.266D+00	-0.858D-01	0.848D+00	0.640D+00	-0.322D+00
0.149D+00	0.100D+01	-0.847D-01	0.806D+00	0.242D+00	0.980D-01	-0.553D+00
0.266D+00	-0.847D-01	0.100D+01	-0.615D+00	0.246D+00	0.655D-01	-0.748D+00
-0.858D-01	0.806D+00	-0.615D+00	0.100D+01	0.444D-02	0.350D-02	-0.721D-02
0.848D+00	0.242D+00	0.246D+00	0.444D-02	0.100D+01	0.251D+00	-0.344D+00
0.640D+00	0.980D-01	0.655D-01	0.350D-02	0.251D+00	0.100D+01	-0.154D+00
-0.322D+00	-0.553D+00	-0.748D+00	-0.721D-02	-0.344D+00	-0.154D+00	0.100D+01

Table 4 continued

ROTATION PARAMETERS CONSTRAINED

NA9 - NAD

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

ΔX METERS	ΔY METERS	ΔZ METERS	ϵ $(x10^{-6})$	θ_z SECONDS	θ_y SECONDS	θ_x SECONDS
-1.91	-20.95	23.51	-4.82	-0.80	0.56	-0.25

VARIANCE - COVARIANCE MATRIX

0.885D+01	0.843D+00	0.449D+01	-0.349D-06	0.990D-06	0.524D-06	-0.665D-06
0.843D+00	0.255D+02	-0.101D+02	0.407D-05	0.355D-06	0.143D-06	-0.874D-06
0.449D+01	-0.101D+02	0.184D+02	-0.291D-05	0.559D-06	0.163D-06	-0.141D-05
-0.349D-06	0.407D-05	-0.291D-05	0.797D-12	-0.122D-13	0.447D-15	0.144D-13
0.990D-06	0.355D-06	0.559D-06	-0.122D-13	0.172D-12	0.299D-13	-0.103D-12
0.524D-06	0.143D-06	0.163D-06	0.447D-15	0.299D-13	0.106D-12	-0.380D-13
-0.665D-06	-0.874D-06	-0.141D-05	0.144D-13	-0.103D-12	-0.380D-13	0.268D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.561D-01	0.352D+00	-0.132D+00	0.802D+00	0.542D+00	-0.432D+00
0.561D-01	0.100D+01	-0.465D+00	0.903D+00	0.169D+00	0.872D-01	-0.334D+00
0.352D+00	-0.465D+00	0.100D+01	-0.759D+00	0.314D+00	0.117D+00	-0.633D+00
-0.132D+00	0.903D+00	-0.759D+00	0.100D+01	-0.329D-01	0.154D-02	0.311D-01
0.802D+00	0.169D+00	0.314D+00	-0.329D-01	0.100D+01	0.221D+00	-0.479D+00
0.542D+00	0.872D-01	0.117D+00	0.154D-02	0.221D+00	0.100D+01	-0.226D+00
-0.432D+00	-0.334D+00	-0.633D+00	0.311D-01	-0.479D+00	-0.226D+00	0.100D+01

Table 4 continued

ROTATION PARAMETERS CONSTRAINED

NA9 - "GC"

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

ΔX METERS	ΔY METERS	ΔZ METERS	ε ($x10^{-6}$)	θ_z SECONDS	θ_y SECONDS	θ_x SECONDS
39.14	-144.40	-202.51	-0.48	-0.37	-0.22	-0.22

VARIANCE - COVARIANCE MATRIX

0.273D+01	-0.130D+01	0.177D+01	-0.349D-06	0.200D-06	0.126D-06	-0.131D-06
-0.130D+01	0.160D+02	-0.838D+01	0.279D-05	0.814D-07	0.406D-07	-0.170D-06
0.177D+01	-0.838D+01	0.769D+01	-0.173D-05	0.993D-07	0.271D-07	-0.257D-06
-0.349D-06	0.279D-05	-0.173D-05	0.522D-12	-0.849D-15	0.119D-14	0.232D-15
0.200D-06	0.814D-07	0.993D-07	-0.849D-15	0.335D-13	0.741D-14	-0.193D-13
0.126D-06	0.406D-07	0.271D-07	0.119D-14	0.741D-14	0.263D-13	-0.912D-14
-0.131D-06	-0.170D-06	-0.257D-06	0.232D-15	-0.193D-13	-0.912D-14	0.500D-13

COEFFICIENTS OF CORRELATION

0.100D+01	-0.197D+00	0.385D+00	-0.292D+00	0.662D+00	0.469D+00	-0.355D+00
-0.197D+00	0.100D+01	-0.755D+00	0.964D+00	0.111D+00	0.625D-01	-0.190D+00
0.385D+00	-0.755D+00	0.100D+01	-0.865D+00	0.195D+00	0.602D-01	-0.415D+00
-0.292D+00	0.964D+00	-0.865D+00	0.100D+01	-0.642D-02	0.102D-01	0.144D-02
0.662D+00	0.111D+00	0.195D+00	-0.642D-02	0.100D+01	0.249D+00	-0.471D+00
0.469D+00	0.625D-01	0.602D-01	0.102D-01	0.249D+00	0.100D+01	-0.251D+00
-0.355D+00	-0.190D+00	-0.415D+00	0.144D-02	-0.471D+00	-0.251D+00	0.100D+01

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2.4 Free Adjustment of a Geometric Global Satellite Network (Solution MPS7)

2.41 Introduction

The basic purpose of this experiment was to compute reduced normal equations from the observational data of several different systems described below to combine them eventually with the normal equations of the Wild BC-4 observations taken in the DOD/DOC cooperative worldwide geodetic satellite program and provide station coordinates from a single least squares adjustment. The solution described in this paper is a partial one obtained without the use of the BC-4 data. The observational systems combined were the Baker-Nunn simultaneous camera observations from the SAO worldwide network; the C-Band range observations from the NASA network; the MOTS and PC-1000 optical observations in North America; miscellaneous camera observations in Europe which were included in the SAO 69 solution; and, lastly, a group of optical observations where Baker-Nunn cameras observed simultaneously with MOTS and/or PC-1000 cameras in the previously mentioned group.

2.42 Description

Smithsonian Data

A set of optical observations were obtained from the Smithsonian Astrophysical Observatory. These included 14,356 simultaneous observations from 28 stations in the SAO 69 Network. For each observation the track angle was provided along with the uncertainties along and across the track. The variances and covariances, in terms of right ascension and declination, were computed as described in [Girnius and Joughin, 1968].

MOTS and PC-1000 Data

The set of optical observations used here were the same as those used in the NA 6 adjustment described in [Mueller, et al., 1969]. The observations had been previously screened and a set of reduced normal equations, referenced to the North American Datum, obtained.

In the meantime [Vincent, et al., 1971] published a geoidal map based on gravimetric and satellite data. By an iterative procedure a new solution was computed which constrained the new undulations, and thus a set of geocentric coordinates were obtained. With these coordinates as initial values, but with the original set of

observations, new reduced normal equations were computed to be used in the solution described in this paper.

C-Band Observations

The C-Band solution is a least squares adjustment of the range observations from twenty-eight C-Band radar stations operated by NASA in a worldwide network, which resulted in distances between the stations and a set of coordinates of the stations on the SAO C-6 ellipsoid along with their standard deviations [Brooks and Leitao, 1970]. Upon request, NASA/Wallops Island kindly sent us the correlation matrix for these solutions, which enabled us to reconstruct the full variance-covariance matrix.

Some of the stations in this adjustment could be related through ground triangulation to nearby Baker-Nunn, MOTS or PC-1000 cameras, thus the interstation distances provided indirectly the scale of the solution. The C-Band data was treated as though they were length observations between the stations and developed a program that computed the normal equations that would correspond to these length-observations utilizing also the reconstructed variance-covariance matrix.

The computed lengths are listed in Table 1.

Table 1

Stations	Length (m)
Merritt Island (4082) to Pretoria (4050)	10,909,592
Merritt Island (4082) to Kauai (4742)	7,362,142
Merritt Island (4082) to Bermuda (4740)	1,593,106
Merritt Island (4082) to Grand Turk (4081)	1,230,691
Merritt Island (4082) to Antigua (4061)	2,288,026
Kauai H.I. (4742) to Vandenberg AFB (4280)	3,977,684

Mixed Optical Observations

We also received from the NASA, National Space Science Data Center a magnetic tape containing records of optical observations on GEOS-I from November of 1965 to August of 1966. These included 2322 simultaneous observations between Baker-Nunn cameras, MOTS cameras and PC-1000 cameras located on and around the North American continent.

It being intended to combine the normal equations developed from the above observations with a set of normal equations developed from a much larger set of SAO, MOTS and PC-1000 described above, observations the possibility of duplicating observations had to be considered.

In the case of the majority of the MOTS and PC-1000 and all of the Baker-Nunn observations, the few duplicated observations were overwhelmed by the large number of other observations at these stations. However, a number of MOTS and PC-1000 stations in the Caribbean area contributed only a few observations to the North American normal equations. Any duplicated observations here would have had an inordinate effect. Therefore, all such observations were eliminated.

2.43 Constraints

Inner Adjustment Constraints (Free Adjustment)

The large number of optical observations effectively determined the orientation of the total network while the C-Band observations provided a scale. Only the origin remained undetermined. To define the origin of the system in its most favorable position (from the error propagation point of view) we imposed "Inner Adjustment Constraints" compelling the trace of the variance-covariance matrix to be a minimum [Blaha, 1971].

Length Constraints

The C-Band observations described earlier introduced scale into our adjustments. They also provided much needed extra connections from Africa across the Atlantic and to the Caribbean Islands, and the length Kauai to Vandenberg Air Force Base greatly strengthened the geometry in the western United States.

In addition to the C-Band scale we also introduced a weighted

chord length constraint between Homestead, Florida and Greenbelt, Maryland derived from updated Cape Canaveral datum coordinates of these two stations determined from the high precision geodimeter traverse in the eastern United States.

Height Constraints

At all stations, a weighted height constraint was imposed. The heights above mean sea level were obtained from [NASA, 1971] and to these, the undulations referred to the SAO 69 ellipsoid were added. The undulations were determined from a number of sources. Between North America and Europe the geoid of [Vincent, et al., 1971] was used. In this report, the undulations of some stations were also tabulated (computed). These tabulated values were constrained with weights corresponding to a standard deviation of 3 m. Other station undulations were interpolated from the geoid map itself and, allowing for interpolation errors, received assigned standard deviations of 5 m except in those areas near the Caribbean where, because of large geoidal gradients, a standard deviation of 8 m was estimated. For stations in other parts of the world (not covered by the above geoid map) the undulations were obtained from the SAO 69 geoid map, and standard deviations from 8 m to 15 m were assigned depending upon the number of gravity measurements available in the surrounding area. All heights constrained (H) are shown in Table 2.

These height constraints, which are in effect independent observations, provided a valuable strengthening of an otherwise weak geometric network. A test adjustment was run (MPS 9) in which all previously described constraints were held except the height constraints and in this adjustment the final standard deviations of the coordinates were more than doubled and at poorly determined stations more than tripled.

Relative Position Constraints

These weighted constraints were used to tie together the C-Band radar stations with nearby camera stations through the connecting triangulation, and also helped to connect the Baker-Nunn stations with nearby MOTS and/or PC-1000 stations.

In every case, Cartesian coordinate differences were computed on the local datum and the weights determined from standard deviations computed from a formula given in [Simmons, 1950]. This estimate was used in all cases except between Merritt Island and Jupiter, Florida, where the uncertainty was estimated to be

Table 2

Height Constraints and Undulations

(all units in meters)

Number	Station	Constraints		N	
		H	σ	MPS7	[Vincent et al., 1971]
1021	Blossom Pt., Md.	- 20	3	- 30	- 26
1022	Fort Myers, Fla.	- 13	3	- 18	- 18
1030	Goldstone, Cal.	902	3	- 27	- 27
1032	St. John's, Nswf.	82	5	12	14
1033	Fairbanks, Alaska	188	15	4	
1034	E. Grand Forks, Minn.	238	3	- 15	- 18
1042	Rosman, N.C.	887	3	- 24	- 22
3106	Antigua, W.I.	- 37	3	- 41	- 39
3334	Stoneville, Miss.	20	5	- 20	- 19
3400	Colorado Springs, Col.	2173	5	- 8	- 11
3401	Bedford, Mass.	63	5	- 28	- 20
3402	Semmes, Alabama	55	3	- 24	- 18
3404	Swan Island	31	15	- 38	
3405	Grand Turk, B.I.	- 29	3	- 39	- 31
3406	Curacao, N. Antilles	- 19	8	- 29	- 26
3407	Trinidad, T. & T.	221	8	- 59	- 34
3648	Hunter AFB, Georgia	- 12	3	- 25	- 24
3657	Aberdeen, Md.	- 20	3	- 26	- 26
3861	Homestead, Fla.	- 22	3	- 24	- 22
3902	Cheyenne, Wyo.	1872	5	- 11	- 10
3903	Herndon, Va.	142	5	- 33	- 26
4082	Merritt Island, Fla.	- 12	3	- 27	- 23
4280	Vandenberg AFB, Cal.	91	3	- 30	- 32
4050	Pretoria, S.A.	1604	6	- 1	
4742	Kauai, H.I.	1157	9	- 4	
7036	Edinburg, Texas	48	3	- 11	- 12
7037	Columbia, Mo.	249	3	- 20	- 24
7039	Bermuda	- 5	3	- 37	- 36
7040	San Juan, P.R.	9	3	- 41	- 41
7043	Greenbelt, Md.	27	3	- 29	- 26

Table 2 (continued)

Number	Station	Constraints		N	
		H	σ	MPS7	[Vincent et al., 1971]
7045	Denver, Col.	1767	3	- 13	- 13
7072	Jupiter, Fla.	- 10	3	- 26	- 24
7075	Sudbury, Canada	251	3	- 30	- 31
7076	Kinsgton, Jamaica	423	3	- 22	- 23
8009	Delft, Holland	72	3	48	47
8010	Zimmerwald, Swiss.	957	3	51	54
8011	Malvern, England	165	5	57	52
8015	Haute Provence, Fr.	702	3	59	
8019	Nice, France	432	3	45	55
8030	Meudon, France	214	5	47	
9001	Organ Pass, N.M.	1633	3	- 16	- 18
9002	Pretoria, S.A.	1564	6	0	
9004	San Fernando, Spain	81	3	45	55
9005	Tokyo, Japan	99	6	41	
9006	Naini Tal, India	1874	8	- 45	
9007	Arequipa, Peru	2477	9	19	
9008	Shiraz, Iran	1588	10	- 20	
9009	Curacao, N. Antilles	- 19	5	- 29	- 26
9010	Jupiter, Fla.	- 9	3	- 27	- 24
9011	Villa Dolores, Arg.	618	8	6	
9012	Maui, Hawaii	3036	9	19	
9021	Mt. Hopkins, Ariz.	2362	3	- 37	- 22
9028	Addis Ababa, Ethiopia	1911	10	52	
9029	Natal, Brazil	37	10	- 12	
9031	Comodoro Rivadavia, Arg.	215	15	- 9	
9051	Athens, Greece	242	5	46	54
9091	Dionysos, Greece	454	3	78	54
9424	Cold Lake, Canada	684	6	- 30	
9425	Edwards AFB, Cal.	756	3	- 24	- 28
9426	Harestua, Norway	622	3	45	46
9427	Johnston Island	17	10	28	
9431	Riga, Latvia	32	3	22	24
9432	Uzhgorod, USSR	236	3	45	47

one part in 750,000. The relative constraints used and their weights ($1/\sigma^2$) are all given in Table 3.

2.44 The Adjustment

The four sets of normal equations (See Section 2.42), and the previously explained constraint equations were added together and a single solution was obtained for the combined systems.

We decided to run three different adjustments to investigate the effects of the constraints we were using: MPS 7 was ultimately chosen as the best adjustment. It contained all the constraints previously explained, inner adjustment plus height constraints. MPS 8 included the height constraints but without inner adjustment. MPS 9 was run with inner adjustment constraints but without holding the heights.

After MPS 7 was run, we immediately computed the undulations (N) at selected stations and compared them with the values given in [Vincent, et al., 1971]. This comparison is given in Table 2. There are some discrepancies, but generally the fit is good, indicating that despite the free adjustment, the height constraints had held (thus our origin is reasonably close to the center of mass).

The results of the MPS 7 adjustment are tabulated in Appendix 1. The number of degrees of freedom was 10586; the quadratic sum of all the residuals 12201; and the standard deviation of unit weight 1.07.

2.45 Comparisons with other Solutions

Table 4 summarizes the transformation parameters (systematic differences) between the MPS 7 coordinates and those published in [Gaposchkin and Lambeck, 1970], and in [Marsh, et al., 1971], for the global network and for both the European and American nets. Two sets of parameters are listed. The first was obtained through the assumption that only translations exist between the sets of coordinates. In the second solution, the rotations were first computed through direction cosines independent of translations and scale factor. Subsequently the general seven-parameter transformation was carried out with the three rotation parameters constrained with their variance-covariances obtained in the direction cosine solution. Appendix 2 gives the general solution and variance-covariance and correlation coefficient matrices obtained in each case.

Table 3

Relative Position Constraints

Distances	Relative Coordinates (meters) ΔX	ΔY	ΔZ	Weights $(1/\sigma^2)$		
4082 Merritt Island to 9010 Jupiter, Florida	- 65710.36	62288.42	137731.50	130.3	145.2	29.7
4280 Vandenberg AFB to 9113 Rosamund, California	- 221861.70	103220.29	- 27546.03	0.2	0.6	3.5
4050 Pretoria, S.A. to 9002 Olifantsfontein, S.A.	- 4499.99	10094.48	1602.05	39.2	13.3	155.5
4742 Kauai, Hawaiian Island to 9012 Maui, Hawaiian Island	- 77906.67	349733.48	145327.36	0.9	0.1	0.4
3406 Curacao PC-1000 9009 Curacao Baker Nunn	- 10.00	3.00	28.00	135194.0	673180.0	34257.0
7072 Jupiter MOTS 40 9010 Jupiter Baker Nunn	- 14.81	1.98	7.84	80051.0	1170662.0	187019.0
4081 Grand Turk C-Band 3405 Grand Turk PC-1000	928.47	1760.31	3352.71	321.6	147.0	58.0
4740 Bermuda C-Band 7039 Bermuda MOTS 40	674.25	- 700.53	- 1476.11	492.6	468.2	173.3
4061 Antigua C-Band 3106 Antigua PC-1000	- 245.92	- 359.44	- 514.23	1890.5	1139.7	707.0

Table 4

Transformation Parameters

		GLOBAL		EUROPE			NORTH AMERICA		
		MPS7-GSFC	MPS7-SAO	MPS7-GSFC	MPS7-SAO	MPS7-ED50	MPS7-NAD	MPS7-GSFC	MPS7-SAO
No. Stations		15	20	11	9	11	28	16	15
Wt. Factor*		25.0	1.42	18.0	0.86	1.08	0.67	17.24	0.36
3 Parameter Transf.	ΔX (m)	-19.3 ±2.6	-12.1 ±2.4	- 2.8 ±2.5	- 3.5 ±2.6	- 90.6 ±3.2	- 39.6 ±1.26	-18.7 ±1.2	-12.6 ±1.3
	ΔY (m)	0.6 ±2.9	- 5.4 ±2.6	-17.5 ±2.7	- 9.7 ±3.2	-138.6 ±3.5	142.9 ±1.04	-18.1 ±1.4	-12.5 ±1.2
	ΔZ (m)	29.7 ±2.9	29.4 ±2.5	31.1 ±2.7	49.7 ±2.9	- 85.5 ±3.3	201.8 ±1.17	22.5 ±1.4	20.5 ±1.3
7 Parameter** Transf.	ΔX (m)	-10.6 ±2.9	- 4.3 ±2.6	-35.8 ±15.3	-45.2 ±17.9	-115.3 ±13.7	- 44.8 ±3.1	3.3 ±3.6	- 7.5 ±3.6
	ΔY (m)	6.5 ±3.6	0.5 ±3.2	-49.5 ± 8.4	36.6 ±12.2	-165.4 ±10.4	147.5 ±4.6	-35.2 ±6.7	-50.2 ±5.9
	ΔZ (m)	32.8 ±3.4	25.4 ±3.0	134.5 ±15.5	19.4 ±18.0	- 89.1 ±12.8	225.1 ±4.1	39.4 ±5.9	40.5 ±5.3
	θ_Z (")	0.49 ±0.06	0.30 ±0.04	-1.58 ±0.22	1.20 ±0.31	-0.64 ±0.39	-0.58 ±0.09	0.51 ±0.10	0.11 ±0.10
	θ_Y (")	0.29 ±0.06	0.53 ±0.04	-3.35 ±0.24	-0.30 ±0.35	-0.63 ±0.51	0.48 ±0.08	0.27 ±0.09	0.08 ±0.10
	θ_X (")	- 0.40 ±0.07	- 0.14 ±0.04	0.03 ±0.22	-1.04 ±0.35	0.56 ±0.34	-0.75 ±0.10	-0.12 ±0.16	0.21 ±0.15
	ϵ ($\times 10^{-6}$)	0.96 ±0.56	1.11 ±0.50	-7.33 ±3.06	7.87 ±3.62	3.45 ±1.53	-1.50 ±0.80	-3.18 ±1.13	-6.76 ±0.99

* Weight Factor = $P_{MPS7}/P_i = \sigma_i^2/\sigma^2_{MPS7}$

** Rotation Parameters Constrained

APPENDIX 1.

(all coordinates in meters)

STATION NUMBER -	1021	BLOSSOM PT. MD.	ELLIPSOID
	X	Y	Z
PREL. COORD. -	1118002.4168	-4876323.0294	3943045.8335
CORRECTIONS -	14.3103	-6.9144	-50.3748
ADJ. COORD. -	1118016.7271	-4876329.9438	3942995.4587
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	12.313516	1.496271	-0.506484
	1.496271	9.574184	4.554761
	-0.506484	4.554761	11.640591

STATION NUMBER -	1022	FORT MYERS FLA.	ELLIPSOID
	X	Y	Z
PREL. COORD. -	807831.3284	-5651992.6336	2833574.2911
CORRECTIONS -	14.6979	-3.8427	-43.3542
ADJ. COORD. -	807846.0263	-5651996.4763	2833530.9369
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	8.464229	0.425609	-0.775519
	0.425609	4.691764	2.508412
	-0.775519	2.508412	8.352738

STATION NUMBER -	1030	GOLDSTONE CAL.	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-2357271.1364	-4646320.9594	3668373.0248
CORRECTIONS -	13.2372	-10.9745	-46.2868
ADJ. COORD. -	-2357257.8992	-4646331.9338	3668326.7380
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	36.984467	-11.738343	4.245114
	-11.738343	12.954854	1.754896
	4.245114	1.754896	12.444189

STATION NUMBER -	1032	ST. JOHN'S NEWF.	ELLIPSOID
	X	Y	Z
PREL. COORD. -	2602652.1506	-3419240.9226	4697706.7911
CORRECTIONS -	31.1331	15.0817	-36.9271
ADJ. COORD. -	2602683.2837	-3419225.8409	4697669.8640
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	1576.740956	1616.226552	425.109336
	1616.226552	2246.928027	569.830226
	425.109336	569.830226	199.539150

STATION NUMBER - 1033	FAIRBANKS ALASKA		ELLIPSOID
	X	Y	Z
PREL. COORD. -	-2299282.0027	-1445689.3178	5751856.7179
CORRECTIONS -	-35.9058	129.6797	-28.4177
ADJ. COORD. -	-2299317.9085	-1445559.6381	5751828.3003

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

131.868362	-226.563917	-22.601573
-226.563917	1474.708556	-14.640450
-22.601573	-14.640450	184.008270

STATION NUMBER - 1034	E.GRAND FORKS MIN		ELLIPSOID
	X	Y	Z
PREL. COORD. -	-521731.2847	-4242049.7016	4718785.4161
CORRECTIONS -	19.0951	-13.3933	-41.1352
ADJ. COORD. -	-521712.1896	-4242063.0949	4718744.2809

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

12.316699	-3.443950	-2.388826
-3.443950	11.786131	6.054013
-2.388826	6.054013	10.666619

STATION NUMBER - 1042	ROSMAN N.C.		ELLIPSOID
	X	Y	Z
PREL. COORD. -	647476.3149	-5177936.8829	3656781.8283
CORRECTIONS -	14.6835	-3.9251	-44.9581
ADJ. COORD. -	647490.9983	-5177940.8080	3656736.8703

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

11.146406	-0.127488	-1.203739
-0.127488	6.986618	3.440431
-1.203739	3.440431	10.659820

STATION NUMBER - 3106	ANTIGUA W.I.		ELLIPSOID
	X	Y	Z
PREL. COORD. -	2881825.7087	-5372175.5983	1868614.2894
CORRECTIONS -	11.3426	-1.7385	-42.6865
ADJ. COORD. -	2881837.0513	-5372177.3368	1868571.6009

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

8.619457	2.290235	2.550995
2.290235	8.530703	3.249506
2.550995	3.249506	15.227372

STATION NUMBER - 3334 STONEVILLE MISS. ELLIPSOID

	X	Y	Z
PREL. COORD. -	-85004.8234	-5337967.6543	3493514.3294
CORRECTIONS -	20.1273	-13.3608	-57.2098
ADJ. COORD. -	-84984.6961	-5327991.0152	3493457.1196

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

193.290286	25.418556	16.967818
25.418556	56.074948	43.805831
16.967818	43.805831	65.515000

STATION NUMBER - 3400 COLORADO SPRINGS ELLIPSOID

	X	Y	Z
PREL. COORD. -	-1275219.1542	-4798014.2603	3994281.6200
CORRECTIONS -	-3.1655	-11.9150	-48.6183
ADJ. COORD. -	-1275222.3197	-4798026.1753	3994233.0017

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

100.522156	5.489216	1.453025
5.489216	39.431720	18.067686
1.453025	18.067686	28.139421

STATION NUMBER - 3401 BEDFORD MASS. ELLIPSOID

	X	Y	Z
PREL. COORD. -	1513121.1433	-4463574.7017	4283136.2634
CORRECTIONS -	8.3585	-5.3602	-50.9707
ADJ. COORD. -	1513129.5019	-4463580.0620	4283085.2927

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

14.199741	2.956042	0.642642
2.956042	19.573991	3.808765
0.642642	3.808765	14.258883

STATION NUMBER - 3402 SEMMES ALABAMA ELLIPSOID

	X	Y	Z
PREL. COORD. -	167235.5703	-5481976.7695	3245114.9169
CORRECTIONS -	16.4549	-1.1263	-47.1968
ADJ. COORD. -	167252.0250	-5481977.8958	3245067.7200

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

19.610556	-0.022277	0.718049
-0.022277	8.198014	4.276106
0.718049	4.276106	14.553314

STATION NUMBER - 3404 SWAN ISLAND ELLIPSOID

	X	Y	Z
PREL. COORD. -	642471.8473	-6053959.8531	1895769.1149
CORRECTIONS -	8.3885	22.5348	-55.5610
ADJ. COORD. -	642480.2359	-6053937.3184	1895713.5539

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

26.221598	-1.938513	5.300409
-1.938513	21.003566	0.738638
5.300409	0.738638	27.691902

STATION NUMBER - 3405 GRAND TURK B.I. ELLIPSOID

	X	Y	Z
PREL. COORD. -	1919474.8507	-5621098.3583	2315843.4691
CORRECTIONS -	4.5264	-9.5606	-29.2670
ADJ. COORD. -	1919479.3772	-5621107.9189	2315814.2021

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

10.192175	2.847803	3.290664
2.847803	10.211564	3.581171
3.290664	3.581171	14.553138

STATION NUMBER - 3406 CURACAO N.ANTILLES ELLIPSOID

	X	Y	Z
PREL. COORD. -	2251788.8707	-5816921.9480	1327276.8528
CORRECTIONS -	9.5577	-0.7047	-53.7261
ADJ. COORD. -	2251798.4284	-5816922.6527	1327223.1268

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

11.255735	2.023911	-2.488057
2.023911	8.224379	0.212203
-2.488057	0.212203	16.691533

STATION NUMBER - 3407 TRINIDAD TOBAGO ELLIPSOID

	X	Y	Z
PREL. COORD. -	2279883.6488	-5513555.9595	1181229.6195
CORRECTIONS -	-2.3349	18.3834	-55.1032
ADJ. COORD. -	2279881.3139	-5513537.5761	1181174.5163

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

37.511455	8.989610	-24.418510
8.989610	26.855695	-14.654408
-24.418510	-14.654408	67.395878

STATION NUMBER -	3648	HUNTER AFB	ELLIPSOID
	X	Y	Z
PREL. COORD. -	832544.6662	-5349546.1754	3360560.8129
CORRECTIONS -	13.2768	-1.6816	-41.2852
ADJ. COORD. -	832557.9430	-5349547.8569	3360519.5277

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

19.342401	1.753618	-1.486431
1.753618	11.223273	6.723818
-1.486431	6.723818	20.223318

STATION NUMBER -	3657	ABERDEEN MARYLAND	ELLIPSOID
	X	Y	Z
PREL. COORD. -	1186768.5113	-4785190.1073	4032960.5255
CORRECTIONS -	12.7733	-10.1142	-46.2672
ADJ. COORD. -	1186781.2846	-4785200.2215	4032914.2583

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

13.786196	2.636266	-0.030759
2.636266	12.869306	5.306248
-0.030759	5.306248	12.636535

STATION NUMBER -	3861	HOMESTEAD FLORIDA	ELLIPSOID
	X	Y	Z
PREL. COORD. -	961749.7864	-5679161.0894	2729960.9763
CORRECTIONS -	12.5787	-3.1751	-45.3326
ADJ. COORD. -	961762.4651	-5679164.2645	2729915.6437

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

12.217638	0.680301	-1.556980
0.680301	6.131925	2.148448
-1.556980	2.148448	11.199167

STATION NUMBER -	3902	CHEYENNE WYOMING	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-1234711.0928	-4651223.4264	4174833.6998
CORRECTIONS -	1.3860	-9.8983	-49.1243
ADJ. COORD. -	-1234709.7067	-4651233.3246	4174784.5755

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

39.816633	6.908121	0.003942
6.908121	60.299385	29.336945
0.003942	29.336945	43.841677

STATION NUMBER - 4903 HERNDON P C 1000 ELLIPSOID

	X	Y	Z
PREL. COORD. -	1088973.4482	-4843050.3590	3991809.2580
CORRECTIONS -	2.2497	38.3767	1.9134
ADJ. COORD. -	1088975.6979	-4843011.9823	3991811.1714

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

153.693242	5.908044	-30.155461
5.908044	70.058888	45.604731
-30.155461	45.604731	82.063819

STATION NUMBER - 4050 PRETORIA ELLIPSOID

	X	Y	Z
PREL. COORD. -	5051608.7772	2726595.1416	-2774193.7107
CORRECTIONS -	43.5116	16.5803	97.8050
ADJ. COORD. -	5051652.2869	2726611.7219	-2774095.9057

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

114.707757	174.660988	343.201160
174.660988	657.411642	1049.241343
343.201160	1049.241343	1752.744465

STATION NUMBER - 4061 ETRANT ELLIPSOID

	X	Y	Z
PREL. COORD. -	2881594.3640	-5372524.5483	1868020.3753
CORRECTIONS -	-3.2296	-12.2235	36.9964
ADJ. COORD. -	2881591.1344	-5372536.7718	1868057.3717

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

8.619286	2.290063	2.551892
2.290063	8.531678	3.249712
2.551892	3.249712	15.229010

STATION NUMBER - 4081 ETRGRT ELLIPSOID

	X	Y	Z
PREL. COORD. -	1920401.8659	-5619398.5502	2319115.5044
CORRECTIONS -	5.9831	-39.0582	51.4104
ADJ. COORD. -	1920407.8490	-5619437.6084	2319166.9148

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

10.191646	2.848073	3.300169
2.848073	10.219372	3.580917
3.300169	3.580917	14.557561

STATION NUMBER - 4082 MERRITT ISLAND ELLIPSOID

	X	Y	Z
PREL. COORD. -	910571.7663	-5539109.3182	3017972.2628
CORRECTIONS -	-10.8556	-10.3649	23.4777
ADJ. COORD. -	910560.9107	-5539119.6831	3017995.7405

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

7.363961	0.986981	-0.848562
0.986981	3.846166	2.830556
-0.848562	2.830556	7.762649

STATION NUMBER - 4250 VANDENBERG AFB ELLIPSOID

	X	Y	Z
PREL. COORD. -	-2671859.9380	-4521204.5748	3607505.6743
CORRECTIONS -	-15.2215	-6.9001	-4.5242
ADJ. COORD. -	-2671875.1595	-4521211.4749	3607501.1501

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

14.754655	0.959815	5.748499
0.959815	16.040498	15.904103
5.748499	15.904103	28.962084

STATION NUMBER - 4740 NBER34 ELLIPSOID

	X	Y	Z
PREL. COORD. -	2308892.0318	-4674293.0140	3393084.9134
CORRECTIONS -	-10.4465	-15.2697	30.2645
ADJ. COORD. -	2308881.5853	-4674308.2837	3393115.1779

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

9.303290	-1.343919	-4.421144
-1.343919	9.931386	3.846743
-4.421144	3.846743	15.940009

STATION NUMBER - 4742 KAUAI H.I. ELLIPSOID

	X	Y	Z
PREL. COORD. -	-5543998.0310	-2054549.4059	2387495.2986
CORRECTIONS -	5.5585	18.8806	11.6629
ADJ. COORD. -	-5543992.4725	-2054530.5253	2387506.9616

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

10.298438	8.962982	-3.071354
8.962982	23.169937	-1.895387
-3.071354	-1.895357	32.814527

STATION NUMBER - 7036 EDINBURG TEXAS ELLIPSOID

	X	Y	Z
PREL. COORD. -	-828519.5327	-5657465.9567	2816885.7705
CORRECTIONS -	23.2938	-7.7280	-43.7482
ADJ. COORD. -	-828496.2390	-5657473.6847	2816842.0224

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

14.345140	-0.792797	1.532558
-0.792797	6.287238	2.173500
1.532558	2.173500	11.162002

STATION NUMBER - 7037 COLUMBIA MO. ELLIPSOID

	X	Y	Z
PREL. COORD. -	-191317.0684	-4967282.1892	3983325.7103
CORRECTIONS -	18.1420	-14.2615	-44.1249
ADJ. COORD. -	-191298.9264	-4967296.4507	3983281.5854

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

11.105130	-1.437472	-1.437143
-1.437472	6.799690	3.652438
-1.437143	3.652438	9.149429

STATION NUMBER - 7039 BERMUDA ELLIPSOID

	X	Y	Z
PREL. COORD. -	2308200.5140	-4873603.7626	3394630.8205
CORRECTIONS -	6.8249	-3.9950	-39.5362
ADJ. COORD. -	2308207.3389	-4873607.7576	3394591.2843

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

9.304137	-1.347172	-4.419626
-1.347172	9.929902	3.847707
-4.419626	3.847707	15.934007

STATION NUMBER - 7040 SAN JUAN P.R. ELLIPSOID

	X	Y	Z
PREL. COORD. -	2465036.6657	-5534931.4949	1965585.1070
CORRECTIONS -	8.4647	-9.2015	-38.9374
ADJ. COORD. -	2465045.1304	-5534940.6964	1965546.1696

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

16.731550	4.019746	-6.333528
4.019746	8.094200	0.162968
-6.333528	0.162968	17.420575

STATION NUMBER -	7043	GREENBELT MARYLAND	ELLIPSOID
	X	Y	Z
PREL. COORD. -	1130691.3085	-4831330.1555	3994211.7859
CORRECTIONS -	11.3710	-5.3198	-46.2172
ADJ. COORD. -	1130702.6795	-4831335.4753	3994165.5687
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	13.005325	0.728954	-0.878702
	0.728954	9.231226	2.819598
	-0.878702	2.819598	9.754693

STATION NUMBER -	7045	DENVER COLORADO	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-1240491.2083	-4760227.8963	4049054.2245
CORRECTIONS -	12.9239	-7.1289	-48.8968
ADJ. COORD. -	-1240478.2844	-4760235.0252	4049005.3276
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	19.376489	-3.993436	-0.808182
	-3.993436	10.623798	4.638193
	-0.808182	4.638193	11.860496

STATION NUMBER -	7072	JUPITER FLORIDA	ELLIPSOID
	X	Y	Z
PREL. COORD. -	976243.9724	-5601403.7081	2880316.4199
CORRECTIONS -	12.4810	-2.4230	-44.3242
ADJ. COORD. -	976256.4533	-5601406.1311	2880272.0957
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	7.360116	0.988111	-0.847620
	0.988111	3.841332	2.827789
	-0.847620	2.827789	7.756602

STATION NUMBER -	7075	SUDBURY CANADA	ELLIPSOID
	X	Y	Z
PREL. COORD. -	692592.8703	-4347069.9512	4600546.1501
CORRECTIONS -	20.1092	-8.5585	-41.4684
ADJ. COORD. -	692612.9795	-4347078.5097	4600504.6817
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	17.128149	1.928309	-0.444263
	1.928309	18.292697	9.041386
	-0.444263	9.041386	15.230182

STATION NUMBER -	7076	KINGSTON JAMAICA	ELLIPSOID
	X	Y	Z
PREL. COORD. -	1384136.9074	-5905665.5321	1966616.7114
CORRECTIONS -	14.9259	-3.0748	-42.6771
ADJ. COORD. -	1384151.8333	-5905668.6069	1966574.0342
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	20.315521	1.712389	-7.262447
	1.712389	9.577170	4.381240
	-7.262447	4.381240	28.680015

STATION NUMBER -	8009	DELFT HOLLAND	ELLIPSOID
	X	Y	Z
PREL. COORD. -	3923410.9853	299882.0032	5002944.9891
CORRECTIONS -	-6.0431	11.1291	60.9986
ADJ. COORD. -	3923404.9422	299893.1323	5003005.9877
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	80.021606	17.796768	-54.092364
	17.796768	81.760344	-22.152040
	-54.092364	-22.152040	51.662614

STATION NUMBER -	8010	ZIMMERWALD SWISS	ELLIPSOID
	X	Y	Z
PREL. COORD. -	4331309.9968	567511.0855	4633092.9783
CORRECTIONS -	4.8275	0.4567	50.6166
ADJ. COORD. -	4331314.8243	567511.5422	4633143.5948
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	40.321597	0.392675	-30.694538
	0.392675	53.570309	-5.967214
	-30.694538	-5.967214	35.341992

STATION NUMBER -	8011	MALVERN ENGLAND	ELLIPSOID
	X	Y	Z
PREL. COORD. -	3920177.9934	-134738.0054	5012707.9782
CORRECTIONS -	-15.4531	-38.3874	59.8594
ADJ. COORD. -	3920162.5403	-134776.3928	5012767.8376
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	33.112650	50.920248	-50.242905
	50.920248	221.223325	-41.602759
	-50.242905	-41.602759	59.674643

STATION NUMBER - 8015 HAUTE PROVENCE, FR ELLIPSOID

	X	Y	Z
PREL. COORD. -	4578327.9964	457965.9986	4403179.0029
CORRECTIONS -	-0.3565	-5.9354	47.6872
ADJ. COORD. -	4578327.6399	457960.0632	4403226.6900

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

23.895656	1.970908	-18.396335
1.970908	55.757516	-6.612086
-18.396335	-6.612086	24.495400

STATION NUMBER - 8019 NICE, FRANCE ELLIPSOID

	X	Y	Z
PREL. COORD. -	4579465.9979	586599.0005	4386408.0076
CORRECTIONS -	2.3669	-8.1364	44.4101
ADJ. COORD. -	4579468.3647	586590.8641	4386452.4177

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

21.937494	0.151965	-17.395617
0.151965	49.235576	-5.219443
-17.395617	-5.219443	21.593170

STATION NUMBER - 8030 MEUDON FRANCE ELLIPSOID

	X	Y	Z
PREL. COORD. -	4205683.8651	163731.9484	4776581.8402
CORRECTIONS -	-50.2822	-28.1516	-9.0951
ADJ. COORD. -	4205633.5829	163703.7968	4776572.7451

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

143.494990	33.860690	-103.562842
33.860690	179.540265	-72.296619
-103.562842	-72.296619	124.832169

STATION NUMBER - 9001 ORGAN PASS, N.M. ELLIPSOID

	X	Y	Z
PREL. COORD. -	-1535756.9990	-5166995.9954	3401042.0033
CORRECTIONS -	1.1876	-14.8514	21.0677
ADJ. COORD. -	-1535755.8114	-5167010.8468	3401063.0710

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

21.673452	-5.862359	-1.832076
-5.862359	9.455905	3.064996
-1.832076	3.064996	10.764767

STATION NUMBER -	9002	PRETORIA, S. AFRIC	ELLIPSOID
	X	Y	Z
PREL. COORD. -	5056125.0005	2716511.0017	-2775784.0066
CORRECTIONS -	27.2805	6.2866	86.0435
ADJ. COORD. -	5056152.2810	2716517.2883	-2775697.9630

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

114.682594	174.626037	343.170668
174.626037	657.421236	1049.170272
343.170668	1049.170272	1752.748138

STATION NUMBER -	9004	SAN FERNANDO, SPAI	ELLIPSOID
	X	Y	Z
PREL. COORD. -	5105587.9980	-555228.0010	3769667.0071
CORRECTIONS -	-4.9988	-21.3000	42.4855
ADJ. COORD. -	5105582.9992	-555249.3010	3769709.4926

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

13.185126	-5.541513	-10.721963
-5.541513	116.202408	11.038475
-10.721963	11.038475	17.726254

STATION NUMBER -	9005	TOKYO, JAPAN	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-3946692.9973	3366298.9990	3698832.0038
CORRECTIONS -	16.0480	32.2762	18.7828
ADJ. COORD. -	-3946676.9492	3366331.2752	3698850.7866

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

1610.625412	1339.767243	370.520660
1339.767243	1318.288910	190.148976
370.520660	190.148976	242.705840

STATION NUMBER -	9006	NAINI TAL, INDIA	ELLIPSOID
	X	Y	Z
PREL. COORD. -	1018202.9992	5471102.9962	3109623.0022
CORRECTIONS -	-42.9173	10.3435	38.0793
ADJ. COORD. -	1018160.0819	5471113.3398	3109661.0814

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

332.689704	-121.490997	59.677658
-121.490997	99.358899	-33.691324
59.677658	-33.691324	79.262703

STATION NUMBER -	9007	AREQUIPA, PERU	ELLIPSOID
	X	Y	Z
PREL. COORD. -	1942774.9998	-5804032.0013	-1796933.0040
CORRECTIONS -	-15.7687	-13.8814	45.8596
ADJ. COORD. -	1942759.2311	-5804094.8828	-1796887.1445

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

13.701890	-0.241407	-2.221664
-0.241407	17.849964	-13.113828
-2.221664	-13.113828	77.063559

STATION NUMBER -	9008	SHIRAZ, IRAN	ELLIPSOID
	X	Y	Z
PREL. COORD. -	3376892.9997	4403975.9985	3136250.0045
CORRECTIONS -	-13.3703	2.8709	43.4706
ADJ. COORD. -	3376879.6294	4403978.8695	3136293.4751

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

76.802336	-38.486143	25.409800
-38.486143	76.445280	-19.501727
25.409800	-19.501727	64.682409

STATION NUMBER -	9009	CURACAO, ANTILLES	ELLIPSOID
	X	Y	Z
PREL. COORD. -	2251828.9991	-5816919.9982	1327160.0025
CORRECTIONS -	-20.5707	-6.6544	35.1242
ADJ. COORD. -	2251808.4284	-5816926.6527	1327195.1268

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

11.255734	2.023912	-2.498057
2.023912	8.224379	0.212202
-2.498057	0.212202	16.691522

STATION NUMBER -	9010	JUPITER, FLA.	ELLIPSOID
	X	Y	Z
PREL. COORD. -	976290.9995	-5601398.0027	2880240.0056
CORRECTIONS -	-19.7314	-10.1094	24.2502
ADJ. COORD. -	976271.2681	-5601408.1122	2880264.2559

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

7.350111	0.996111	-0.847619
0.996111	3.841332	2.827789
-0.847619	2.827789	7.756801

STATION NUMBER -	9011	VILLA DOLORES, ARG	ELLIPSOID
	X	Y	Z
PREL. COORD. -	2280588.9984	-4914572.9998	-3355426.0070
CORRECTIONS -	-10.9111	-2.5516	41.6426
ADJ. COORD. -	2280578.0873	-4914575.5514	-3355384.3644
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	16.531580	5.167778	-12.888693
	5.167778	53.070055	-70.680682
	-12.888693	-70.680682	183.816794

STATION NUMBER -	9012	MAUI, HAWAII	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-5466052.9998	-2404282.0002	2242171.0037
CORRECTIONS -	-32.8294	17.0400	8.9532
ADJ. COORD. -	-5466085.8292	-2404264.9602	2242179.9569
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	10.657097	5.881140	-2.808946
	5.881140	21.418478	-4.896343
	-2.808946	-4.896343	30.730210

STATION NUMBER -	9021	MT. HOPKINS, ARIZ.	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-1936781.9960	-5077703.9979	3331916.0047
CORRECTIONS -	-64.3690	27.6680	46.4301
ADJ. COORD. -	-1936846.3650	-5077676.3298	3331962.4348
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	529.632309	-348.185962	-213.696170
	-348.185962	261.116454	173.431425
	-213.696170	173.431425	141.029361

STATION NUMBER -	9028	ADDIS ABABA, ETHIO	ELLIPSOID
	X	Y	Z
PREL. COORD. -	4903750.0002	3965200.9989	963872.0027
CORRECTIONS -	-16.8935	-4.5775	49.6444
ADJ. COORD. -	4903733.1067	3965196.4214	963921.6471
VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION			
	54.159702	-0.030083	-8.875643
	-0.030083	75.127101	-33.846265
	-8.875643	-33.846265	190.710875

STATION NUMBER - 9029 NATAL, BRAZIL ELLIPSOID

	X	Y	Z
PREL. COORD. -	5186461.0031	-3653856.0041	-654325.0004
CORRECTIONS -	9.5709	-16.6133	8.3574
ADJ. COORD. -	5186470.5741	-3653872.6174	-654316.6430

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

146.612546	96.078844	259.384984
96.078844	247.655904	262.105164
259.384984	262.105164	935.850670

STATION NUMBER - 9031 COMODORO RIVADAVIA ELLIPSOID

	X	Y	Z
PREL. COORD. -	1693802.9990	-4112327.9956	-4556649.0032
CORRECTIONS -	-15.5660	-10.1859	-8.5227
ADJ. COORD. -	1693787.4330	-4112338.1816	-4556657.5259

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

92.966936	-49.644253	67.995681
-49.644253	162.291201	-132.630168
67.995681	-132.630168	299.898936

STATION NUMBER - 9051 ATHENS GREECE ELLIPSOID

	X	Y	Z
PREL. COORD. -	4606857.3417	2029885.1016	3903534.3033
CORRECTIONS -	15.8611	28.0189	71.0936
ADJ. COORD. -	4606873.2028	2029713.1205	3903605.3970

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

47.769841	-76.776197	0.122217
-76.776197	332.835451	-52.340421
0.122217	-52.340421	47.224241

STATION NUMBER - 9091 DIONYSOS, GREECE ELLIPSOID

	X	Y	Z
PREL. COORD. -	4595157.0017	2039424.9995	3912650.0080
CORRECTIONS -	-10.6191	4.7407	40.7523
ADJ. COORD. -	4595146.3826	2039429.7402	3912690.7603

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

23.791676	-23.246814	-7.42[illegible]
-23.246814	109.449891	-12.149702
-7.42[illegible]	-12.149702	16.662[illegible]

STATION NUMBER -	9424	COLD LAKE, CANADA	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-1264837.9987	-3466883.9982	5185467.0083
CORRECTIONS -	4.1202	-11.6673	3.7336
ADJ. COORD. -	-1264833.8785	-3466895.6655	5185470.7419

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

27.601371	0.750136	1.095511
0.750136	42.163804	14.351202
1.095511	14.351202	35.662172

STATION NUMBER -	9425	ROSAMOND, CAL.	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-2450010.9973	-4624420.9955	3635035.0032
CORRECTIONS -	-4.8477	-10.3622	12.1509
ADJ. COORD. -	-2450015.8449	-4624431.3577	3635047.1541

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

16.600406	0.007882	6.086793
0.007882	15.353003	15.218732
6.086793	15.218732	28.788095

STATION NUMBER -	9426	HARESTUA, NORWAY	ELLIPSOID
	X	Y	Z
PREL. COORD. -	3121279.9959	592642.9993	5512701.0014
CORRECTIONS -	-13.1041	-16.7612	54.2884
ADJ. COORD. -	3121266.8918	592626.2381	5512755.2898

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

96.845752	21.567709	-55.233439
21.567709	92.663076	-20.241667
-55.233439	-20.241667	44.135516

STATION NUMBER -	9427	JOHNSTON ISL., PAC	ELLIPSOID
	X	Y	Z
PREL. COORD. -	-6007402.0008	-1111858.9984	1825730.0040
CORRECTIONS -	-53.0498	75.4625	-12.4449
ADJ. COORD. -	-6007455.0506	-1111783.5359	1825717.5590

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

191.043567	-183.546727	59.407868
-183.546727	497.723454	-171.506040
59.407868	-171.506040	105.414781

STATION NUMBER - 9431 RIGA LATVIA ELLIPSOID

	X	Y	Z
PREL. COORD. -	3183901.0315	1421447.9236	5322771.9790
CORRECTIONS -	-1.5281	-3.8555	63.5594
ADJ. COORD. -	3183899.5035	1421444.0680	5322835.5384

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

195.748692	-65.070918	-99.977784
-65.070918	73.934484	22.425051
-99.977784	22.425051	64.788047

STATION NUMBER - 9432 UZGHOROD,USSR ELLIPSOID

	X	Y	Z
PREL. COORD. -	3907420.9947	1602397.0020	4763890.0104
CORRECTIONS -	1.2646	1.3643	65.4506
ADJ. COORD. -	3907422.2593	1602398.3662	4763955.4610

VARIANCE-COVARIANCE MATRIX OF THE STATION POSITION

80.693382	-50.515008	-41.444638
-50.515008	94.468279	12.077380
-41.444638	12.077380	38.571770

APPENDIX 2.

ROTATION PARAMETERS CONSTRAINED MPS7 - GSFC (GLOBAL)

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL $(x10^{-7})$	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-10.56	6.47	32.82	5.55	0.49	0.29	-0.40

VARIANCE - COVARIANCE MATRIX

0.846D+01	-0.177D+01	0.106D+01	-0.518D-06	0.291D-06	0.154D-06	-0.178D-07
-0.177D+01	0.127D+02	-0.189D+01	0.114D-05	-0.182D-08	-0.464D-07	-0.176D-06
0.106D+01	-0.189D+01	0.118D+02	-0.693D-06	0.551D-07	-0.126D-06	-0.395D-06
-0.518D-06	0.114D-05	-0.693D-06	0.317D-12	-0.112D-13	-0.597D-14	0.216D-14
0.291D-06	-0.182D-08	0.551D-07	-0.112D-13	0.851D-13	0.619D-14	-0.113D-13
0.154D-06	-0.464D-07	-0.126D-06	-0.597D-14	0.619D-14	0.104D-12	0.173D-13
-0.178D-07	-0.176D-06	-0.395D-06	0.216D-14	-0.113D-13	0.173D-13	0.119D-12

COEFFICIENTS OF CORRELATION

0.100D+01	-0.171D+00	0.106D+00	-0.316D+00	0.343D+00	0.164D+00	-0.177D-01
-0.171D+00	0.100D+01	-0.154D+00	0.568D+00	-0.176D-02	-0.403D-01	-0.143D+00
0.106D+00	-0.154D+00	0.100D+01	-0.358D+00	0.551D-01	-0.114D+00	-0.324D+00
-0.316D+00	0.568D+00	-0.358D+00	0.100D+01	-0.682D-01	-0.328D-01	0.111D-01
0.343D+00	-0.176D-02	0.551D-01	-0.682D-01	0.100D+01	0.656D-01	-0.112D+00
0.164D+00	-0.403D-01	-0.114D+00	-0.328D-01	0.656D-01	0.100D+01	0.156D+00
-0.177D-01	-0.143D+00	-0.324D+00	0.111D-01	-0.112D+00	0.156D+00	0.100D+01

ROTATION PARAMETERS CONSTRAINED MPS7 – SAO (GLOBAL)

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL ($\times 10^{-7}$)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-4.33	0.52	25.41	11.13	0.30	0.53	-0.14

VARIANCE – COVARIANCE MATRIX

0.701D+01	-0.110D+01	0.797D+00	-0.347D-06	0.114D-06	0.121D-06	0.120D-07
-0.110D+01	0.995D+01	-0.200D+01	0.835D-06	0.152D-07	-0.258D-07	-0.987D-07
0.787D+00	-0.200D+01	0.874D+01	-0.692D-06	0.130D-07	-0.690D-07	-0.125D-06
-0.347D-06	0.835D-06	-0.692D-06	0.255D-12	-0.380D-14	-0.219D-14	0.851D-15
0.114D-06	0.152D-07	0.130D-07	-0.380D-14	0.384D-13	0.424D-14	-0.287D-14
0.121D-06	-0.258D-07	-0.690D-07	-0.219D-14	0.424D-14	0.507D-13	0.977D-14
0.120D-07	-0.987D-07	-0.125D-06	0.851D-15	-0.287D-14	0.977D-14	0.465D-13

COEFFICIENTS OF CORRELATION

0.100D+01	-0.132D+00	0.101D+00	-0.259D+00	0.221D+00	0.203D+00	0.211D-01
-0.132D+00	0.100D+01	-0.214D+00	0.524D+00	0.247D-01	-0.363D-01	-0.145D+00
0.101D+00	-0.214D+00	0.100D+01	-0.463D+00	0.224D-01	-0.104D+00	-0.197D+00
-0.259D+00	0.524D+00	-0.463D+00	0.100D+01	-0.384D-01	-0.193D-01	0.781D-02
0.221D+00	0.247D-01	0.224D-01	-0.384D-01	0.100D+01	0.960D-01	-0.679D-01
0.203D+00	-0.363D-01	-0.104D+00	-0.193D-01	0.960D-01	0.100D+01	0.201D+00
0.212D-01	-0.145D+00	-0.197D+00	0.781D-02	-0.679D-01	0.201D+00	0.100D+01

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL ($\times 10^{-7}$)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-35.77	-49.50	134.52	-73.30	-1.58	-3.35	0.03

VARIANCE - COVARIANCE MATRIX

0.234D+03	0.357D+02	0.145D+03	-0.419D-04	-0.489D-06	0.857D-05	-0.187D-05
0.357D+02	0.701D+02	0.952D+01	-0.590D-05	0.602D-05	0.280D-05	-0.642D-05
0.145D+03	0.952D+01	0.241D+03	-0.420D-04	-0.150D-05	-0.108D-04	0.295D-05
-0.419D-04	-0.590D-05	-0.420D-04	0.936D-11	0.134D-12	0.171D-12	-0.159D-13
-0.489D-06	0.602D-05	-0.150D-05	0.134D-12	0.118D-11	0.170D-12	-0.235D-12
0.857D-05	0.280D-05	-0.108D-04	0.171D-12	0.170D-12	0.219D-11	-0.480D-12
-0.187D-05	-0.642D-05	0.295D-05	-0.159D-13	-0.235D-12	-0.480D-12	0.119D-11

COEFFICIENTS OF CORRELATION

0.100D+01	0.279D+00	0.611D+00	-0.894D+00	-0.294D-01	0.378D+00	-0.112D+00
0.279D+00	0.100D+01	0.732D-01	-0.230D+00	0.662D+00	0.226D+00	-0.703D+00
0.611D+00	0.732D-01	0.100D+01	-0.884D+00	-0.887D-01	-0.468D+00	0.174D+00
-0.894D+00	-0.230D+00	-0.884D+00	0.100D+01	0.405D-01	0.378D-01	-0.477D-02
-0.294D-01	0.662D+00	-0.887D-01	0.405D-01	0.100D+01	0.106D+00	-0.199D+00
0.378D+00	0.226D+00	-0.468D+00	0.378D-01	0.106D+00	0.100D+01	-0.297D+00
-0.112D+00	-0.703D+00	0.174D+00	-0.477D-02	-0.199D+00	-0.297D+00	0.100D+01

ROTATION PARAMETERS CONSTRAINED MPS7 - SAO (GLOBAL)

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL $(x10^{-7})$	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-45.23	36.64	19.39	78.68	1.20	-0.30	-1.04

VARIANCE - COVARIANCE MATRIX

0.319D+03	0.783D+02	0.214D+03	-0.588D-04	0.362D-05	0.104D-04	-0.675D-05
0.783D+02	0.150D+03	-0.223D+02	-0.735D-05	0.137D-04	0.112D-04	-0.157D-04
0.214D+03	-0.223D+02	0.324D+03	-0.585D-04	-0.592D-05	-0.116D-04	0.620D-05
-0.588D-04	-0.735D-05	-0.585D-04	0.131D-10	0.145D-12	0.330D-13	0.195D-12
0.362D-05	0.137D-04	-0.592D-05	0.145D-12	0.206D-11	0.110D-11	-0.109D-11
0.104D-04	0.112D-04	-0.116D-04	0.330D-13	0.110D-11	0.248D-11	-0.141D-11
-0.675D-05	-0.157D-04	0.620D-05	0.195D-12	-0.109D-11	-0.141D-11	0.238D-11

COEFFICIENTS OF CORRELATION

0.100D+01	0.358D+00	0.667D+00	-0.911D+00	0.141D+00	0.371D+00	-0.245D+00
0.358D+00	0.100D+01	-0.101D+00	-0.166D+00	0.782D+00	0.580D+00	-0.833D+00
0.667D+00	-0.101D+00	0.100D+01	-0.899D+00	-0.230D+00	-0.410D+00	0.224D+00
-0.911D+00	-0.166D+00	-0.899D+00	0.100D+01	0.279D-01	0.579D-02	0.350D-01
0.141D+00	0.782D+00	-0.230D+00	0.279D-01	0.100D+01	0.490D+00	-0.492D+00
0.371D+00	0.580D+00	-0.410D+00	0.579D-02	0.490D+00	0.100D+01	-0.579D+00
-0.245D+00	-0.833D+00	0.224D+00	0.350D-01	-0.492D+00	-0.579D+00	0.100D+01

ROTATION PARAMETERS CONSTRAINED MPS7 - ED '50

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL ($\times 10^{-7}$)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-113.39	-165.74	-90.77	34.40	-0.66	-0.54	0.55

VARIANCE - COVARIANCE MATRIX

0.766D+02	0.116D+02	0.183D+02	-0.980D-05	-0.244D-05	0.554D-05	-0.145D-05
0.116D+02	0.338D+02	0.155D+02	-0.379D-05	0.239D-05	-0.345D-06	-0.175D-05
0.183D+02	0.155D+02	0.729D+02	-0.981D-05	0.191D-05	-0.500D-05	0.174D-05
-0.980D-05	-0.379D-05	-0.981D-05	0.231D-11	-0.304D-13	-0.295D-13	0.219D-13
-0.244D-05	0.239D-05	0.191D-05	-0.304D-13	0.694D-12	-0.416D-12	0.941D-13
0.554D-05	-0.345D-06	-0.500D-05	-0.295D-13	-0.416D-12	0.119D-11	-0.300D-12
-0.145D-05	-0.175D-05	0.174D-05	0.219D-13	0.941D-13	-0.300D-12	0.501D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.228D+00	0.245D+00	-0.737D+00	-0.335D+00	0.581D+00	-0.233D+00
0.228D+00	0.100D+01	0.312D+00	-0.429D+00	0.493D+00	-0.545D-01	-0.425D+00
0.245D+00	0.312D+00	0.100D+01	-0.756D+00	0.269D+00	-0.537D+00	0.288D+00
-0.737D+00	-0.429D+00	-0.756D+00	0.100D+01	-0.240D-01	-0.178D-01	0.204D-01
-0.335D+00	0.493D+00	0.269D+00	-0.240D-01	0.100D+01	-0.459D+00	0.160D+00
0.581D+00	-0.545D-01	-0.537D+00	-0.178D-01	-0.459D+00	0.100D+01	-0.389D+00
-0.233D+00	-0.425D+00	0.288D+00	0.204D-01	0.160D+00	-0.389D+00	0.100D+01

ROTATION PARAMETERS CONSTRAINED MPS7 - NAD '27

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL $(x10^{-7})$	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-44.75	147.52	225.10	-15.01	-0.58	0.48	-0.75

VARIANCE - COVARIANCE MATRIX

0.960D+01	0.124D+01	0.500D+01	-0.357D-06	0.110D-05	0.601D-06	-0.771D-06
0.124D+01	0.215D+02	-0.644D+01	0.323D-05	0.431D-06	0.161D-06	-0.102D-05
0.500D+01	-0.644D+01	0.171D+02	-0.232D-05	0.638D-06	0.191D-06	-0.155D-05
-0.357D-06	0.323D-05	-0.232D-05	0.637D-12	-0.111D-13	-0.753D-15	0.101D-13
0.110D-05	0.431D-06	0.638D-06	-0.111D-13	0.192D-12	0.353D-13	-0.120D-12
0.601D-06	0.161D-06	0.191D-06	-0.753D-15	0.353D-13	0.121D-12	-0.443D-13
-0.771D-06	-0.102D-05	-0.155D-05	0.101D-13	-0.120D-12	-0.443D-13	0.302D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.865D-01	0.390D+00	-0.144D+00	0.814D+00	0.557D+00	-0.453D+00
0.865D-01	0.100D+01	-0.336D+00	0.874D+00	0.212D+00	0.995D-01	-0.399D+00
0.390D+00	-0.336D+00	0.100D+01	-0.704D+00	0.352D+00	0.132D+00	-0.683D+00
-0.144D+00	0.874D+00	-0.704D+00	0.100D+01	-0.319D-01	-0.271D-02	0.231D-01
0.814D+00	0.212D+00	0.352D+00	-0.319D-01	0.100D+01	0.231D+00	-0.500D+00
0.557D+00	0.995D-01	0.132D+00	-0.271D-02	0.231D+00	0.100D+01	-0.231D+00
-0.453D+00	-0.399D+00	-0.683D+00	0.231D-01	-0.500D+00	-0.231D+00	0.100D+01

ROTATION PARAMETERS CONSTRAINED MPS7 - GSFC (N.A.)

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL ($\times 10^{-7}$)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
3.29	-35.20	39.38	-31.77	0.51	0.27	-0.12

VARIANCE - COVARIANCE MATRIX

0.129D+02	0.139D+01	0.533D+01	-0.314D-06	0.135D-05	0.890D-06	-0.839D-06
0.139D+01	0.444D+02	-0.119D+02	0.660D-05	0.431D-06	0.172D-06	-0.217D-05
0.533D+01	-0.119D+02	0.352D+02	-0.445D-05	0.647D-06	0.306D-06	-0.325D-05
-0.314D-06	0.660D-05	-0.445D-05	0.128D-11	-0.379D-14	-0.565D-14	0.244D-15
0.135D-05	0.431D-06	0.647D-06	-0.379D-14	0.236D-12	0.412D-13	-0.124D-12
0.890D-06	0.172D-06	0.306D-06	-0.565D-14	0.412D-13	0.196D-12	-0.578D-13
-0.839D-06	-0.217D-05	-0.325D-05	0.244D-15	-0.124D-12	-0.578D-13	0.636D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.579D-01	0.250D+00	-0.773D-01	0.776D+00	0.560D+00	-0.293D+00
0.579D-01	0.100D+01	-0.302D+00	0.876D+00	0.133D+00	0.583D-01	-0.408D+00
0.250D+00	-0.302D+00	0.100D+01	-0.664D+00	0.225D+00	0.117D+00	-0.687D+00
-0.773D-01	0.876D+00	-0.664D+00	0.100D+01	-0.690D-02	-0.113D-01	0.270D-03
0.776D+00	0.133D+00	0.225D+00	-0.690D-02	0.100D+01	0.192D+00	-0.321D+00
0.560D+00	0.583D-01	0.117D+00	-0.113D-01	0.192D+00	0.100D+01	-0.164D+00
-0.293D+00	-0.408D+00	-0.687D+00	0.270D-03	-0.321D+00	-0.164D+00	0.100D+01

ROTATION PARAMETERS CONSTRAINED MPS7 - SAO (N.A.)

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

DX METERS	DY METERS	DZ METERS	DL $(\times 10^{-7})$	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-7.51	-50.15	40.47	-67.64	0.11	0.08	0.21

VARIANCE - COVARIANCE MATRIX

0.131D+02	0.187D+01	0.553D+01	-0.309D-06	0.138D-05	0.982D-06	-0.908D-06
0.187D+01	0.345D+02	-0.801D+01	0.502D-05	0.496D-06	0.260D-06	-0.189D-05
0.553D+01	-0.801D+01	0.277D+02	-0.337D-05	0.645D-06	0.348D-06	-0.267D-05
-0.309D-06	0.502D-05	-0.337D-05	0.974D-12	-0.594D-15	-0.139D-14	-0.780D-14
0.138D-05	0.496D-06	0.645D-06	-0.594D-15	0.236D-12	0.516D-13	-0.128D-12
0.982D-06	0.260D-06	0.348D-06	-0.139D-14	0.516D-13	0.206D-12	-0.743D-13
-0.908D-06	-0.189D-05	-0.267D-05	-0.780D-14	-0.128D-12	-0.743D-13	0.534D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.883D-01	0.291D+00	-0.866D-01	0.786D+00	0.598D+00	-0.344D+00
0.883D-01	0.100D+01	-0.260D+00	0.867D+00	0.174D+00	0.974D-01	-0.442D+00
0.291D+00	-0.260D+00	0.100D+01	-0.649D+00	0.252D+00	0.146D+00	-0.695D+00
-0.866D-01	0.867D+00	-0.649D+00	0.100D+01	-0.124D-02	-0.310D-02	-0.109D-01
0.786D+00	0.174D+00	0.252D+00	-0.124D-02	0.100D+01	0.234D+00	-0.361D+00
0.598D+00	0.974D-01	0.146D+00	-0.310D-02	0.234D+00	0.100D+01	-0.224D+00
-0.344D+00	-0.442D+00	-0.695D+00	-0.109D-01	-0.361D+00	-0.224D+00	0.100D+01

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2.5 Determination of Transformation Parameters with Constraints

The relationship between any two geodetic reference systems would generally consist of seven parameters - three translations (dX, dY, dZ) between the two origins, three rotations (ω, ψ, ϵ) of the Euler's angle type between the two sets of axes and the scale factor (ΔS), if any.

A general transformation for the seven parameters is given below [Badekas, 1969]:

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} \equiv \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 - \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_1 - \begin{bmatrix} 1 & \omega & -\psi \\ -\omega & 1 & \epsilon \\ \psi & -\epsilon & 1 \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}_1 - \Delta S \begin{bmatrix} U \\ V \\ W \end{bmatrix}_1 = 0 \qquad (1)$$

where ω, ψ and ϵ correspond to rotations about Z, Y and X axes respectively - the positive direction of rotations taken in counterclockwise mode from UVW-system to XYZ-system. The above equation can then be further modified as below:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_X \\ v_Y \\ v_Z \\ v_U \\ v_V \\ v_W \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & -U & -V & W & 0 \\ 0 & -1 & 0 & -V & U & 0 & -W \\ 0 & 0 & -1 & -W & 0 & -U & V \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta S \\ \omega \\ \psi \\ \epsilon \end{bmatrix} + \begin{bmatrix} X-U \\ Y-V \\ Z-W \end{bmatrix} = 0 \qquad (2)$$

However, in the above transformation, if the geodetic reference systems are properly defined for Laplace condition (parallelism of minor axis of the reference ellipsoid and earth's rotation axis) the three rotations arising out due to the improper orientation of the system are generally never more than a few seconds of arc while translations may amount up to 200 to 300 meters. Thus due to the presence of high correlation between the rotations and translations, satis-

factory estimates for rotations are difficult in a combined general transformation.

An alternative method separates the determination of the rotations independent of the translations and the scale factor [Bursa, 1966]. The mathematical model is as follows:

$$
\begin{aligned}
&T_{ik}^{(1)} - T_{ik}^{(2)} + \omega - \epsilon \cos T_{ik}^{(1)} \tan \delta_{ik}^{(1)} + \psi \sin T_{ik}^{(1)} \tan \delta_{ik}^{(1)} = 0 \\
&\delta_{ik}^{(1)} - \delta_{ik}^{(2)} + \epsilon \sin T_{ik}^{(1)} + \psi \cos T_{ik}^{(1)} = 0
\end{aligned}
\qquad (3)
$$

where T_{ik} and δ_{ik} are defined as the geodetic hour angle and declination of the (i-k)th direction of the observed point at kth station and the observer at ith station. The indexes (1) and (2) denote the two systems with transformation proceeding from system #1 to system #2.

If we take A_{ik}, B_{ik}, C_{ik} as the direction cosines of the (i-k)th direction, R_{ik} as the length, then for the first system we get

$$
\begin{aligned}
A_{ik} &= \frac{U_k - U_i}{R_{ik}} = \frac{\Delta U_{ik}}{R_{ik}} \\
B_{ik} &= \frac{V_k - V_i}{R_{ik}} = \frac{\Delta V_{ik}}{R_{ik}} \\
C_{ik} &= \frac{W_k - W_i}{R_{ik}} = \frac{\Delta W_{ik}}{R_{ik}}
\end{aligned}
\qquad (4)
$$

and

$$
\begin{aligned}
T_{ik} &= -\arctan \frac{B_{ik}}{A_{ik}} \\
\delta_{ik} &= \arctan \left[\frac{C_{ik}}{(A_{ik}^2 + B_{ik}^2)^{\frac{1}{2}}} \right]
\end{aligned}
\qquad (5)
$$

In the above relations (3, 4 and 5) the elements of translation do not enter the picture and a similar set of relations as per (4) and (5) above can be established for the second system.

The equation (3) then can be written as below:

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_T \\ v_\delta \end{bmatrix}_{ik} + \begin{bmatrix} 1 & \sin T_{ik}^{(1)} \tan \delta_{ik}^{(1)} & -\cos T_{ik}^{(1)} \tan \delta_{ik}^{(1)} \\ 0 & \cos T_{ik}^{(1)} & \sin T_{ik}^{(1)} \end{bmatrix} \begin{bmatrix} \omega \\ \phi \\ \epsilon \end{bmatrix} + \begin{bmatrix} T_{ik}^{(1)} - T_{ik}^{(2)} \\ \delta_{ik}^{(1)} - \delta_{ik}^{(2)} \end{bmatrix} = 0 \qquad (6)$$

Using the variance-covariances matrices ΣX and ΣU in respect of ith and kth points for the XYZ and UVW systems, the variance-covariance matrices $\Sigma_{T\delta}$ were computed for the two systems through propagation of errors as per the following relation [Uotila, 1967]:

$$\Sigma_{T\delta}^{(1)} = G \begin{vmatrix} \Sigma U_i & 0 \\ 0 & \Sigma U_k \end{vmatrix} G' \qquad (9)$$

where

$$G = \begin{bmatrix} \frac{\partial T_{ik}^{(1)}}{\partial U_i} & \frac{\partial T_{ik}^{(1)}}{\partial V_i} & \frac{\partial T_{ik}^{(1)}}{\partial W_i} & \frac{\partial T_{ik}^{(1)}}{\partial U_k} & \frac{\partial T_{ik}^{(1)}}{\partial V_k} & \frac{\partial T_{ik}^{(1)}}{\partial W_k} \\ \\ \frac{\partial \delta_{ik}^{(1)}}{\partial U_i} & \frac{\partial \delta_{ik}^{(1)}}{\partial V_i} & \frac{\partial \delta_{ik}^{(1)}}{\partial W_i} & \frac{\partial \delta_{ik}^{(1)}}{\partial U_k} & \frac{\partial \delta_{ik}^{(1)}}{\partial V_k} & \frac{\partial \delta_{ik}^{(1)}}{\partial W_k} \end{bmatrix}$$

and

$$\frac{\partial T_{ik}}{\partial U_i} = \frac{-\partial T_{ik}}{\partial U_k} = -\frac{\Delta V_{ik}}{\Delta U_{ik}^2 + \Delta V_{ik}^2}$$

$$\frac{\partial T_{ik}}{\partial V_i} = \frac{-\partial T_{ik}}{\partial V_k} = -\frac{\Delta U_{ik}}{\Delta U_{ik}^2 + \Delta V_{ik}^2}$$

$$\frac{\partial T_{ik}}{\partial W_i} = \frac{-\partial T_{ik}}{\partial W_k} = 0$$

$$\frac{\partial \delta_{ik}}{\partial U_i} = \frac{-\partial \delta_{ik}}{\partial U_k} = \frac{\Delta U_{ik}\, \Delta W_{ik}}{R_{ik}^2 \sqrt{\Delta U_{ik}^2 + \Delta V_{ik}^2}}$$

$$\frac{\partial \delta_{ik}}{\partial V_i} = \frac{-\partial \delta_{ik}}{\partial V_k} = \frac{\Delta V_{ik}\ \Delta W_{ik}}{R_{ik}^2 \sqrt{\Delta U_{ik}^2 + \Delta V_{ik}^2}}$$

$$\frac{\partial \delta_{ik}}{\partial W_i} = \frac{-\partial \delta_{ik}}{\partial W_k} = \frac{-\sqrt{\Delta U_{ik}^2 + \Delta V_{ik}^2}}{R_{ik}^2}$$

$$R_{ik}^2 = \Delta U_{ik}^2 + \Delta V_{ik}^2 + \Delta W_{ik}^2$$

Obtaining similarly $\Sigma_{T\delta}^{(2)}$, the combined variance-covariance matrix to be used with the equation (6) would be given by

$$\begin{bmatrix} \Sigma_{T\delta}^{(2)} & 0 \\ 0 & \Sigma_{T\delta}^{(1)} \end{bmatrix} .$$

The above transformation model was used to study the relationship between various datums with the recent free adjustment of a Geometric Global Satellite Network, Solution MPS7 ([Mueller, Whiting, 1972] and Section 2.4). Firstly, the three rotations were obtained independently of the translations with their variance-covariance matrices. Secondly, using the same set of common points a general transformation for seven parameters (including the three translations and the scale factor) was obtained utilizing the rotations in a constrained solution. This transformation was carried out in three broad groups based on the area-wise study i.e., global, European and North America, with the following datums:

(i) Goddard Space Flight Center Reference System (GSFC) [Marsh, Douglas and Klosko, 1971].

(ii) Smithsonian Astrophysical Observatory's Global Reference System (SAO) [Gaposchkin and Lambeck, 1970].

(iii) European Datum 1950 (ED50).

(iv) North American Datum 1927 (NAD).

Table 1 gives the results for three rotations as obtained independently of translations, while Table 2 gives the constrained solution for seven parameters. Table 3 shows the results of a non-constrained general transformation for a comparative study.

The comparison shows that the constrained solutions show an overall improvement in all the transformations. The standard deviations in all the cases are smaller and the variances of unit weight show a better fit in the constrained solution as against the non-constrained transformation.

Table 1

Transformation Parameters From Direction Cosines

		GLOBAL		EUROPE			NORTH AMERICA		
		MPS7-GSFC	MPS7 - SAO	MPS7-GSFC	MPS7 - SAO	MPS7-ED50	MPS7 - NAD	MPS7-GSFC	MPS7-SAO
No. Stations		15	20	11	9	14	28	16	15
Wt. Factor*		25.0	1.42	18.0	0.86	1.08	0.67	17.24	0.36
3 Parameter Transf.	$\theta_Z('')$	0.46 ±0.07	0.31 ±0.04	-1.54 ±0.24	1.20 ±0.31	-0.67 ±0.19	-0.60 ±0.10	0.53 ±0.11	0.13 ±0.11
	$\theta_Y('')$	0.32 ±0.08	0.54 ±0.05	-3.36 ±0.33	-0.27 ±0.34	-0.52 ±0.25	0.47 ±0.08	0.28 ±0.10	0.09 ±0.10
	$\theta_X('')$	-0.41 ±0.08	-0.14 ±0.05	0.002 ±0.24	-1.08 ±0.33	0.55 ±0.16	-0.75 ±0.13	-0.15 ±0.18	0.20 ±0.17
	σ_0^2	1.39	1.04	0.63	0.35	1.62	3.09	1.14	1.71

*Weight Factor = $p_{MPS7}/p_i = \sigma_i^2/\sigma_{MPS7}$

Table 2

Transformation Parameters

(Solution with Weighted Constraints on Rotation Parameters)

		GLOBAL		EUROPE			NORTH AMERICA		
		MPS7-GSFC	MPS7-SAO	MPS7-GSFC	MPS7-SAO	MPS7-ED50	MPS7-NAD	MPS7-GSFC	MPS7-SAO
No. Stations		15	20	11	9	14	28	16	15
Wt. Factor*		25.0	1.42	18.0	0.86	1.08	0.67	17.24	0.36
7 Parameter Transf.	ΔX(m)	-10.56 ±4.10	-4.33 ±2.98	-35.77 ±12.58	-45.23 ±11.41	-113.39 ±11.41	-44.78 ±5.12	3.29 ±3.70	-8.01 ±4.37
	ΔY(m)	6.47 ±5.01	0.52 ±3.55	-49.50 ± 6.87	36.64 ± 7.79	-165.74 ± 7.60	147.44 ±7.64	-35.20 ±6.88	-47.87 ±6.93
	ΔZ(m)	32.82 ±4.82	25.41 ±3.33	134.52 ±12.74	19.39 ±11.44	-90.77 ±11.11	225.16 ±6.82	39.38 ±6.19	38.95 ±6.37
	θ_Z (″)	0.49 ±0.08	0.30 ±0.05	-1.58 ± 0.18	1.20 ± 0.18	-0.66 ± 0.22	-0.58 ±0.15	0.51 ±0.10	0.11 ±0.11
	θ_Y (″)	0.29 ±0.09	0.53 ±0.05	-3.35 ± 0.25	-0.30 ± 0.20	-0.54 ± 0.29	0.48 ±0.12	0.27 ±0.09	0.09 ±0.11
	θ_X (″)	-0.40 ±0.09	-0.14 ±0.05	0.03 ± 0.18	-1.04 ± 0.20	0.55 ± 0.19	-0.75 ±0.19	-0.12 ±0.17	0.20 ±0.18
	ϵ ($\times 10^{-6}$)	0.96 ±0.80	1.11 ±0.57	-7.33 ± 2.30	7.87 ± 2.30	3.44 ± 1.98	-1.52 ±1.31	-3.18 ±1.17	-6.40 ±1.20
	σ_0^2	1.98	1.27	0.67	0.40	1.70	2.73	1.07	1.51

*Weight Factor = $p_{MPS7}/p_i = \sigma_i^2/\sigma_{MPS7}^2$

Table 3

Transformation Parameters

		GLOBAL		EUROPE			NORTH AMERICA		
		MPS7-GSFC	MPS7 - SAO	MPS7-GSFC	MPS7 - SAO	MPS7-ED50	MPS7 - NAD	MPS7-GSFC	MPS7-SAO
No. Stations		15	20	11	9	14	28	16	15
Wt. Factor*		25.0	1.42	18.0	0.86	1.08	0.67	17.24	0.36
7 Parameter Transf.	ΔX(m)	-9.69 ±5.17	-5.79 ±3.80	-33.35 ±18.71	-52.71 ±20.21	-115.34±18.63	-42.47±10.80	-0.70± 9.45	-9.62± 9.41
	ΔY(m)	6.27 ±5.42	0.61 ±3.87	-58.84 ±17.57	25.70 ±28.85	-165.38±14.16	147.25± 9.99	-38.26±10.04	-48.22± 9.92
	ΔZ(m)	32.82 ±6.08	25.75 ±3.89	133.95 ±21.45	25.64 ±21.81	- 89.09±17.44	244.75±11.88	34.79±12.40	38.46±11.67
	ϑ_Z (″)	0.58 ±0.19	0.24 ±0.13	-1.84 ± 0.50	1.13 ± 0.73	- 0.64± 0.53	-0.51± 0.34	0.38± 0.29	0.05± 0.28
	ϑ_Y (″)	0.20 ±0.20	0.47 ±0.15	-3.28 ± 0.72	-0.60 ± 0.82	- 0.63± 0.68	0.51± 0.27	0.24± 0.28	0.08± 0.26
	ϑ_X (″)	-0.38 ±0.23	-0.15 ±0.15	0.22 ± 0.51	-0.62 ± 0.79	0.56± 0.45	-0.73± 0.43	0.06± 0.46	0.22± 0.44
	ϵ ($\times 10^{-6}$)	0.93 ±0.82	1.15 ±0.58	-7.41 ± 2.70	8.10 ± 2.51	3.45± 2.10	-1.53± 1.34	-3.18± 1.18	-6.40± 1.20
	σ_0^2	2.13	1.28	0.75	0.46	1.85	2.81	1.13	1.63

*Weight Factor = $p_{MPS7}/p_1 = \sigma_1^2/\sigma_{MPS6}^2$

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2.6 The Impact of Computers on Surveying and Mapping

Keynote Address Presented by Ivan I. Mueller at the Annual Meeting of the Permanent Committee, International Federation of Surveyors, Tel Aviv, May 29-June 3, 1972

Most keynote speakers usually start with the statement that they are honored and privileged for the opportunity to present their views. I will not be an exception to this custom because I truly feel honored and privileged being selected by the organizing committee to deliver one of the keynote addresses at this meeting. Over the years, the International Federation of Surveyors has consistently sponsored a full range of valuable meetings dedicated to the examination of important problems facing this very diversified profession. Among the most innovative of the convocations called have been those associated with the meetings of the permanent committee.

What then is the purpose of a keynote address? It is generally understood to have a double aim. The first is to arouse unity and enthusiasm in the audience. But I need not concern myself with that, because I am sure that everyone here is equally excited at the potential of computer usage in surveying and mapping and at the new vistas visible on the horizon of this ancient profession. The other purpose of a keynote address is to present the issues inherent in the theme of the meeting. I shall try to present these issues, first as they are related to the computers, then how these machines affected traditional areas within our profession, what new exciting areas came into existence because the machines happened to be around, and finally what are those new vistas just around the horizon which are visible to this observer.

The Computer

When the computer was invented in the fifties, there was a great diversity of opinion on its usefulness, from skeptics who proclaimed it a toy to the more adventuresome prophets who predicted phenomenal growth and widespread application. Reflecting now on some of those early prophecies, it is obvious that they were vague about specific applications, real benefits, actual costs and the technological advances required to make the computer practical. And yet, the

usefulness has outstripped the dreams of the most adventuresome prophets. Undoubtedly, most people's ideas (not ours of course) about computers are associated with erroneous electricity or bank accounts, T V science fiction, moon shots or tax collection. Contrary to these beliefs, computers have a great deal more to offer. They work as calculators too, as repositories of information, as controllers, as aids to decision making in such contexts as banking systems, reservation systems, air and road traffic control. The use of computers as simulators is an application which is growing in importance: Examples include training astronauts, observing the effects of car crashes, playing war games instead of real ones, and business strategies. Computers have also penetrated the field of art to the dismay of some of us: Attempts have been made, with varying success, to use the computers as language translators, as writers of poetry and prose, as producers of visual art, to create ballet routines, and both write and synthesize music. There is plenty of scope here for those of us who enjoy a debate guaranteed to have no conclusive outcome. On the serious side, because of its varied applications, the computer demands from society, including the surveyors, decisions as important as any it has made, certainly as important as those forced on our predecessors by the industrial revolution. It is sad that the level of discussion, even in some "professional" circles, has so far been so puerile, to understanding of the issues so limited and so inadequate.

With this in mind, allow me, in a few minutes, to review the progress over the past two decades to see how the use of computers has developed and then to examine current trends.

The first decade of computer development, in the 1950's, saw the use of machinery largely as an aid to scientific research; many research projects in physics, chemistry and engineering demand elaborate calculations - the design of an aircraft wing or engine, for instance, or the design of a nuclear reactor. As a matter of fact, there is one project - atomic bomb development - which has always demanded more and more calculations in order to progress with as little

testing as possible. It is easier and also rather more socially acceptable to simulate an explosion on a computer, however large and expensive, than to explode a live bomb. This one use played an important part in the development of very large and very fast computers during that first decade. It was not until after 1960 that such machines found their way into other than atomic research laboratories. The second decade of computer development, in the 1960's, saw the development of the computer as an electronic office, a data handler and processor. The computers initially used in this era were designed not as calculating engines for scientific use but to make the processing of card files cheaper and easier. The jobs being done were those which are carried out within the administrative and accounting departments of a business. Such jobs placed more emphasis on the storage capacity available in the machine than on its calculating speed - in contrast to the research applications in the first decade. As the users became more confident in and more used to computers, new applications appeared using both the calculating capacity of the machinery and its data handling capabilities.

In looking back, it becomes relatively easy to separate the demarcation points between post generations of computers. Historically, these have occurred following advances in hardware technology: vacuum tubes for the first generation around 1950/51, transistors for the second (between 1958 - 60), and integrated transistor circuits for the third between 1963 and 1965. Lately, however, the introduction of many other new features - in peripherals, communications, remote terminals, operating systems, and the like - have made the distinction between the generations increasingly fuzzy. We have now passed the eve of the fourth generation computers which is best characterized by the ability to provide information which is constantly on the tap. In other words, while the roles of the first three generations were computations, data and information processing, the current generation also provides on-line information. The rapid evolution through the fourth generation - spurred on primarily by the immense proliferation of minicomputers - is underway and one can now begin to imagine the hardware and software components

which will characterize the fifth generation projected to be born between 1975 and 1978.

I will not elaborate on the technical aspects of these future babies of the computer industry. Let me just say that these new machines are being viewed as man's "intelligent" assistants. Many of them will be portable, hand carried or in the car and in the home, that can be plugged into telephone and electric outlets or even carry their own power supply. This will tie the computer completely to the telecommunications systems, allowing the computer to 'remote' its power to where it is needed. Indeed, the telephone will become probably the most widely used terminal of the 1970's - incorporating voice output and touch tone input. Such an availability of computer power can have nothing less than an immense impact on society, greater perhaps even than the impact television has had.

New major innovations are likely to occur also in the software area. For instance, the cost of programming, which has been held almost constant (per line of code) throughout the past three generations, should be reduced by more than a factor of ten in fourth generation systems. This should come as a direct result of interactive programming using time shared facilities. A further factor of ten reduction in costs can be expected with the fifth generation. With the remote terminal and the packaged programs (to which I will return a little later) will come a truly conversational use of the computers. Many such systems are now being designed and use languages suitable even for the non-professional. By the end of the fifth generation - by the early 1980's - literally anyone will be able to use a computer and many programs should be available for helping us perform our daily tasks. Computers and terminals could then become as common as telephone and television today.

In passing through the second and third generations of computers there was approximately a four fold increase in the number of computers in use per generation. Throughout the 1960's there was a ten fold increase. Assuming that these trends

continue, then by 1975 - at the onset of the fifth generation - there will be more than 200,000 computers in use around the world. By 1980 there could be over 500,000. But if we count the remote terminals, then these numbers grow by a further factor of ten. Moreover, if we include all the telephones used for remote access to computers, then practically everyone with a telephone will have access to a computer by 1980.

What are the uses of all these computers? In addition to applications in our own profession there are of course countless applications. Let me select for illustration probably the most sophisticated one, the applications in management science:

The major object of modern computer applications in this field is the setting up of a computerized data base to enable better analysis to be made of alternative uses of resources. At present, many important decisions are taken on inadequate data or on information which is out of date. In a stable and well-established business this may be of little consequence, but for firms in rapidly changing markets or involved in rapid growth or technological change, timeliness of data can be vital. Rapid and convenient access to the data base is therefore required, and it is necessary that the whole system be designed so it can react to the users urgent demands. Modern computer techniques enable the user to converse with the computer over a terminal. The user can ask questions of the computer, which can then, by questioning the user, ellicit further information to retrieve the answers required from its memory. In this way, the data base can be searched, and the result of a requested analysis can be made instantaneously available.

The nature and complexity of the analysis required may differ considerably, so that it would be inefficient to have the most powerful processor tied up wholly with one user. The equipment needed to implement such an enquiry system is thus, not one computer, but a collection of units, some of which are devoted mainly to manipulating data, some to the calculations needed for analysis of the data, some

to up-dating the files as new data arrives, and some to conversing with the users. As the users and the data sources may be physically distributed over a wide geographic area, the whole complex must be connected by communications channels, and thus, becomes a computer-network. At present such networks are being built for several applications. Several already exist - for instance, to carry out airline and hotel reservations on a world-wide basis. Others are being installed to link hospitals into the data base containing information on patients, availability of beds, etc. There is no intrinsic reason why, in due course, single overall systems should not serve the needs of all the users in any technical or geographical group desired. Several computer bureau operators with machines in different countries are planning to link their machinery so they can work on whichever machine is most readily available or most economic at the time. Such arrangements could well form the basis for an international computer-network.

The establishment of such a network naturally will contain some inherent dangers for the individual, primarily related to his status within the community, who can be affected without his knowledge. In order to bring about beneficial applications, the computer must have data - not only about money and materials and the rest of the physical environment in which we live, but also about people and their attitudes and circumstances. Until recently the clerical effort needed to cross-reference all these files has fortunately been prohibitive. But once these data find their way into a computer system, cross-connections could be made in a matter of second. Thus, on applying for an insurance benefit you might find the amount of your last unpaid parking fine deducted automatically, or perhaps find yourself arrested to answer a charge of speeding. Would we be happy under an efficient tyranny - one in which every movement and action of the citizen was recorded, analyzed, cross-checked instantaneously and no incident, no matter how trivial, ever forgotten? Yet, such is the mechanism we now have the capacity to create. It is not a far stretch of the imagination from here to see that Orwell's <u>1984</u> predictions on surveillance could also be fulfilled

and on schedule.

It is not, of course, the computer itself which creates social problems, but the human beings into whose hands it is placed. The computer is a tool and it can be used or abused by man at his discretion. Compared with such tools as nuclear energy, the computer does seem to possess more potential for good than harm.

Whether this picture appeals to you or frightens you, I have no way of knowing. A recently published book entitled Future Shock, concerned itself with the plight of modern man attempting to cope with "an environment so ephemeral, unfamiliar and complex as to threaten millions with adaptive breakdown." The book is an indication of the apprehension with which some people view the future and it is worthwhile for those of us who are contributing agents of technological evolution to do some hard thinking about where we are going - to alleviate the fears of some and help all prepare for the coming advance in technologies.

Let us now take a look at how the availability of the generations of computers affected surveying and mapping. Obviously, this review will have to be a selective and a subjective one. I will be able to mention only the most spectacular examples and only those which are likely to be in the interest of this convocation, and of course, only those which are in my area of competence.

The Shape of the Earth and its Gravity Field

I should make it clear at the outset, that I am not concerned with local irregularities in the earth's surface, the mountains and the valleys. I shall be discussing the mean sea level surface of the earth, carried through under the land, the surface usually called the geoid. This geoid, being a surface on which the potential of the earth's gravity field is constant, will, at the same time serve as a pictorial representation of the variations in the gravity field of the earth as well.

In the United States, a historical review on the subject in "which shape the

earth is in," probably would start from the time when the Declaration of Independence from England was signed in 1776. In this country, however, one is obliged to start with the prehistoric man, who, if he thought about the subject at all, presumably concluded that, apart from local oddities like rocks or mountains, the earth was flat. This is also the view held today by the Flat-Earth Society, also in England.

The idea of a nearly-spherical earth was surprisingly late in becoming established, or so it seems to us, with the advantage of hindsight. Neither the Babylonians nor the Egyptians favored this idea, and the credit goes to Pythagoras and his school in the sixth century B.C. I should add that the idea was derived not from observations but from their conviction that the sphere was "the perfect" shape. Three-hundred years later Eratosthenes did more than adopt the idea, he actually measured the earth's circumference, using the propagation velocity of a camel caravan as his scale.

It was not until the seventeenth century that the shape of the earth was improved upon. The first indication that the earth may be flattened at the poles was obtained in 1672 by Jean Richer's French expedition to South America, where he found that his pendulum clock, accurate in Paris, was loosing time at Cayenne. First numerical estimates on the flattening came from Newton in his "Principia" published in 1687, but practical measurements to establish the value of the flattening were made by the Cassinis, who measured arc length in France, and who came to the conclusions that the earth was flattened indeed, but not at the poles, but rather at the equator, thus, it looked like an egg or a lemon. This was in 1720, and fierce controversy followed: Was the earth flattened or elongated at the poles? Who was right, Newton or the Cassinis? The French Academy sent the two famous expeditions of Maupertuis to Lapland, and La Condamine's to Peru. After ten years of labor and an equal number of years spent in quarrels, the conclusions tended to confirm Newton's idea, and Voltaire congratulated the expeditions saying, "You have successfully flattened

the poles and the Cassinis." Not much happened after the regarding the shape of the earth, until the middle of the present century, when first analysing gravity measurements on a global basis, and after 1957, analysing the orbits of artificial satellites, a complete new picture of the earth's shape emerged. These analyses, of course, were made possible only because by that time, the computers came into existence.

I shall not describe how from the perturbations to satellite orbits, caused by the various possible oddities in the earth's shape, these oddities can be determined. It should suffice to say that a new value for the flattening has emerged, indicating that the earth's equatorial diameter exceeds the polar diameter by 42.77 km, which is a full 170 meters different from the previously adopted value. This difference may not seem much for most of us, but it is important for the geophysicist, who may conclude that the earth's interior has great strength, and the assumption that it can be treated as if it were a fluid, an assumption which in the past, was widely made, is illegitimate.

The more accurate value for the flattening is, however, only a very small part of the information obtained from satellites. Without going into technicalities, let me simply illustrate the improvement by the fact, that in the pre-satellite era, the shape of the earth and its gravity field was described by four basic parameters, while today, the number of known parameters exceeds two-hundred and fifty. This new information pictorially represented as the aforementioned geoid above the ellipsoid shows that the most prevailing features are the healthy depression around the South Pole, a bulge south of the equator, and also around the North Pole, indicating, in the language of the press, that the earth is "pear-shaped." This discovery came as a relatively great surprise to most of us, but it should have been no surprise to Christopher Columbus, who gave it as his opinion "that it has the shape of a pear that is very round, except where the stem is, which is higher..." Other important features are the depression south of India, (113 m), the elevation near New Guinea, (81 m), and the elevation centered in England

and the south Atlantic.

To sum up, satellites and the computers have brought us from the earth of 1957, which was merely a sphere flattened at the poles, and flattened by the wrong amount, to a complicated figure which when seen in the round looks perhaps like a potato with dips and humps all over it.

By-products of this satellite-orbit analysis are the coordinates of the tracking stations with respect to the center of the earth. In the pre-satellite era, such information, which is vital in relating the numerous geodetic systems of the world, practically did not exist. Today, geocentric coordinates are known for about 150 stations fairly evenly distributed around the globe.

Satellites also help in mapping, as geometric triangulation points in the sky in connection with the method called:

Satellite Triangulation or Trilateration

This method found wide range applications in connecting another 150 - 200 tracking stations in the relative sense both on a continental and on a global basis. Better known projects in this category are the programs under the coordination of the Eastern and Western European Subcommissions for Satellite Triangulation of the International Association of Geodesy; the U.S. National Geodetic Satellite Program now in its final stages, including observations by the Smithsonian Astrophysical Observatory, The National Geodetic Survey, (formerly Coast and Geodetic Survey), NASA and various other agencies; the French coordinated ISAGEX Program; other French works in southern Europe and northern Africa; and some other local national network developments in North and South America.

I will not attempt to offer you a glimpse at the software used in the calculations related to satellite geodesy, mainly because some of them are rather lengthy. The fact that some of these programs took 100 man years to develop is an indication not only of the complexity of the problem, but also of the need for better

programming methods. Clearly, when one needs to work with several ten-thousand observations in order to determine several hundred unknown quantities, like station coordinates, gravitational parameters, and at the same time, attempts to recover at least some of the systematic errors burdening the observations, the computer software and hardware will have to be impressive indeed.

This leads us to an application where the impact of computers is and will probably be the greatest both in its economical aspects and also in the number of people affected. This application is generally known in surveying circles as:

Adjustment Computations

Adjustment in the surveying and mapping terminology is the method used to derive unique and "best" values for parameters from redundant measurements of those parameters, or parameters related to them by a known mathematical relationship. It is a device which should be used by everyone in the profession involved in the evaluation of survey data from leveling to satellite laser ranging or, from cadaster surveys to lunar mapping. The fundamentals of this science were laid down by Karl Friedrich Gauss in the eighteenth century at the age of 18. Every geodesist and photogrammetrist of note since then, has contributed to the literature by refining (or confusing) some aspects of the topic.

Without going again into the technical details to the extent possible, let me remind you that in the pre-computer era, up to the early fifties, one did not enter lightly into an adjustment computation; one looked very closely at the model; one checked and double-checked the input data, and in very special circumstances, one might undertake the extra computations necessary to check the possible correlations between the unknown parameters, or to compute the error ellipses for certain selected points of special interest. In other words, it was not practically feasible to put adjustment computations on a sound statistical basis. The number of unknown parameters was also limited, since the computations

for one medium-sized network (50 - 100 unknowns) were likely to require several man-months of time, thus, it was a very expensive undertaking indeed. The use of statistical methods for planning a network to make sure that it is the most economical and most favorable from the point of view of the propagation of errors was almost out of the question because of the costs involved. For this reason, in a given country, very few organizations were doing adjustment computations.

Today, thanks to the computers, this situation is part of history. Very large numbers of organizations are doing adjustment computations using computer programs, either developed by themselves or procured from other organizations. These programs are (or should be) based on correct statistical theory and techniques, and running them, even with a very large number of unknowns, costs very little.

Advances in this regard were most spectacular in that part of the mapping industry which deals in photogrammetry, where the wide applications of aerial triangulation or analytical photogrammetry using block adjustment techniques with a great number of unknowns is part of the daily routine. Another spectacular area where adjustment computations are routinely used to full capacity is satellite geodesy, where the number of unknown parameters, mostly highly correlated, and to be adjusted for in one huge simultaneous adjustment, may reach several thousand.

It is interesting to note that a significant number of rather sophisticated "package programs" written for different purposes, like aerial triangulation, horizontal control, satellite triangulation or orbit determination, have been widely distributed and used by a great number of organizations other than those who designed the programs. It is a small step from here to arrive to the point, where the average surveyor can pick up his phone and dial the computer or go to his remote terminal, specify his object, read the input data in the specified manner, and receive his results with all the statistical trimmings faster and cheaper

than he ever dreamed of. He has a powerful design tool at his command; he can now make full use of law of error propagation and optimize any system he is designing; he can build in constraints; he can test options and find the option that meets his specifications with the least effort and cost. At the conclusion of the project, he can do an evaluation and test the assumptions that it was necessary to make about his instruments. If data from a variety of sensors have been combined in an adjustment, he can test the distribution of residuals for normality; he can test his mathematical model, his weighing procedure. In theory, this always has been possible, but until modern computer facilities became available, it was out of the question as a regular tool.

Equipment Oriented Areas

There are also equipment oriented areas where the availability of the generations of computers (directly part of, or tied to, the sensor-system) affected surveying and mapping. To mention a few, let me start with the AN/USQ-28 Mapping and Surveying System, which comprises the most advanced group of equipment integrated to collect accurate raw data for mapping purposes. It was specifically designed to acquire photography suitable for 1:50,000 scale topographic mapping in areas where ground control is insufficient. The system is built into a Boeing 707 aircraft and consists of precision mapping cameras, an inertial navigation system, electronic distance measuring equipment, a terrain profile recorder, and other auxiliary equipment. All data, with the exception of the photography, are recorded on magnetic tapes for direct input into digital equipment to speed the data reduction process. It is a pity that as of this moment, the system is not operating because there seems to be lack of money to pay for the operation of the aircraft (for gasoline!).

Another example is the progress that has been made in automated computation equipment. These computer-driven machines use image sensing and correlation techniques to produce horizontally correct images while simultaneously detecting

and recording height information. This equipment is supposed to reduce map compilation time by 75%.

Another development of significant interest is the automatic or semi-automatic coordinate readers. This equipment is designed to measure, for example, the coordinates of star images on photographic plates obtained for astronomic or satellite geodetic applications. The instruments have a pre-programming feature which moves a detecting head to the approximate location of each required star image. The detection head then centers itself precisely over a star image, at which point the coordinates are measured and recorded on punch cards for input in the computer program.

Another and rather esoteric computer application in this equipment oriented category is the Apollo mapping system for accurate lunar mapping. The main purpose of the system is similar to that of the USQ-28 mapping system mentioned earlier, i. e., to provide maps in areas where ground control is insufficient. The lunar orbiter and Apollo programs through Apollo 14 have produced phenomenal photography to support landing site selection and surface operations. However, the new metric camera system which was flown first on Apollo 15, then on Apollo 16 and which will be also on board the last manned flight to the moon, offers an order of magnitude improvement towards lunar mapping, the determination of the lunar gravity field, and of the motion of the moon in space. It is again a pity that the system is included only in the last three missions, and was left out from the previous seven missions. Of course, the astronauts on Apollo 7 - 10 were rather busy preparing the landing of Neal Armstrong on Apollo 11, but only NASA knows why the system was not flown on Apollo 12 - 14. The area coverage would have been certainly better.

This system consists of three cameras, a laser altimeter and timing equipment. The first camera is a 3-inch metric mapping camera which photographs the lunar surface while the second stellar camera built into the same housing takes simultaneous pictures of the star field just above the lunar horizon to aid the deter-

mination of the orientation of the mapping camera. The laser altimeter is synchronized to fire simultaneously and provides the distance from the camera to the lunar ground for each photograph. All this information together with the earth-based tracking data should give sufficient information on the position and orientation of the mapping camera (to about 2.5 m relative). The third 24 inch panoramic camera provides very high resolution photographs (2 m at the nominal 110 km altitude).

As I mentioned, the main application of the system is to establish geodetic control on the moon and provide maps for the areas covered. In addition to these, information is expected on the rotation of the moon about its axis, commonly known as the phenomena of libration. The data will also be analyzed in conjunction with the laser distances measured between earth-based observatories and the reflectors placed on the moon surface by Apollos 11, 14, and 15, and Luna 17. This combination of data should be most helpful to improve on the lunar ephemeris, i.e., on the knowledge of the relative motion of the moon around the earth, which lately seems to be part of geodesy also.

The Future

From here, there is only a short step into the future. What will the next decade bring? I already described what is expected from the computers and how they will change the job of the surveyor in the adjustment area. Let us see briefly that in addition to the routine mapping and surveying activities, what miracles the surveyor is to perform during the next decade or so. First of all, he is going to get some new customers: the geophysicists and the oceanographers. He will need new tools, because their demand for a full magnitude and better positions (from 10 m to 1 m to 10 cm) than what is available today exceeds present capabilities. Most of these instruments are already in the development stage and undoubtedly will be ready for applications in the not too distant future. Let us take a quick look at these machines:

First of all, existing laser distance measuring devices will be improved to the point where the only factor limiting the accuracy of the observations will be the uncertainties in tropospheric propagation, which is expected to be reduced to about 6 cm (from the present 15 - 30 cm).

On the radio frequency systems with prospects of 1 m or better accuracy, Very Long Base-Line Interferometry (VLBI) seems to offer the greatest versatility. This technique depends upon local frequency standards of high quality - preferably hydrogen masers - at two or more radio antennae separated by distances on the earth as great as allowed by the common visibility of a radio source, like a quasar or a water vapor source. The frequency standards provide time references for magnetic tape recordings of signals from these galactic energy sources. The tapes are later correlated at a central computing facility, and the time difference for arrival of the same wave is determined. From this, it will be possible to calculate the distance between the two antennae to an accuracy of about 15 cm and the direction between them to about 0.001 arc second, provided that the position of the energy source is known.

Satellite to satellite (range rate) tracking also will offer substantial advantages over current techniques limited by our dirty window towards space, the atmosphere. Very high satellites will track a low satellite continuously through the vacuum of the universe with very high precision.

Such continuous tracking technique, coupled with the so-called "drag-free" satellite, will further improve our knowledge of the gravity field of the earth and the geoid. The essential element of such a system is an unsupported mass contained in a spherical shell. A control system in the satellite senses motions of the shell relative to the proof mass and actuates small thrusters that force the shell to follow the proof mass without touching it. Hence, the proof mass follows an orbit influenced only by gravitational force.

Improvement in the knowledge of the gravity field, the shape of the geoid is also expected through the satellite to ocean radar altimeters, measuring contin-

uously the distance between sea level and the satellite. The first of these devices will be flown probably in 1974 on an experimental basis.

From these new instruments, a wealth of information will be made available to the earth scientists, who, in turn, will be able to produce unpredictable but certainly substantial advances about the rotational motions of the earth, tide interactions, temporal variations in the gravity field, continental drift and other large scale deformations of the earth crust and mantle. The interactions of these motions and deformations appear to be responsible for a wide variety of effects, including large earthquakes, mountain building, generation of tsunamis (tidal waves), and confinement of nearly all active volcanoes to only a few narrow belts. The satellite born radar altimeter eventually will provide valuable oceanographic information on tides, storm surges, general ocean circulation, and other dynamical processes affecting sea level.

Most of these problems are global in nature, thus, require observations globally distributed. The interaction between the several dynamic subsystems of the earth demands coordination of the observations. Hence, for maximum effectiveness, technological integration and international cooperation are essential to a progressive investigation of these topics.

Is the International Federation of Surveyors willing and ready to participate in this cooperation? It it ready and willing to take this challenge and serve the new customers?

What else is coming? - Automated data banks with national and international links.

- Automated data reduction systems

- Remote sensing satellites for environmental monitoring, ocean sensing and for land use and resources management, producing 15,000,000 bits of information per second - equivalent to an Encyclopedia Britannica every couple of minutes. We certainly will be able to verify the conjuncture that as civilized man evolved from his primitive ancestry, he developed an appetite for large masses of data, recording observations about his individual or collective activities

with ever greater precision and detail.

Concluding Remarks

On the surface, it would seem that surveying presents no serious issues as a technology. It is a useful tool in the service of mankind and extends the capabilities of science. Unlike some technologies, surveying does not pollute. On the contrary, it may help to preserve the quality of the environment. It is not likely to be wasteful economically. Instead, it could stimulate and guide resource development as well as scientific research in the earth sciences. What is more, it has some popular attributes. It requires a private and public sector team effort, and is multi-disciplinary as well as multi-institutional and multi-national.

But, if we were to conclude from such reasoning that no major issues are involved, we would be badly mistaken. The issues are not technological, but sociological. In my view, they effect the unity of the profession of surveying and mapping.

Let me quote a recent editorial from the transactions of the AGU on the "Surveyor Geodesist":

> "For over two thousand years, the land surveyor and the astronomer often joined by the mathematician, collaborated in the development of geodetic science. This symbiotic relationship, which reached its zenith in the last three hundred years, resulted in inferences of geodetic significance from observational data and also led to the establishment of the science on a rigorous mathematical foundation. The surveyor, to some degree and to a limited accuracy, participated in this development in the small; but, today he is severely hampered by the restrictive limits to his data base, by the limited scope of his observing instruments and computing methods, and, in no small way, by the deemphasis in surveying education at the university level. In addition, photogrammetric methods and, in more recent times, developments in space technology have made enormous inroads into his areas of competence. In fact, the phenomenal geodetic fallout from the space program has so obscured the place of the surveyor in the geodetic scheme of things that there is a tendency to downgrade his continued vital contribution to the science. Hence, more and more the average surveyor finds himself outside the geodetic mainstream, relegated to a supporting role as a provider of cadastral and lower order engineering data.

The new team combines the expertise of the mathematician, the physicist, and the space scientist. From space-oriented observations, this group of scientists has obtained data in regions inaccessible to the surveyor and has obtained results that the geodesist using classical techniques could never hope to achieve. As the space scientist refines his measurements and increases his sampling rate, thereby providing more precise data at ever decreasing wave lengths, the geodesist finds that among many applications he can support the oceanographer in resolving ocean surface problems; the tectono-physicist and the seismologist in measuring continental drift and crustal movement; and the astronomer in determining polar motion and variations in earth rotation.

Will this expanded geodetic role further divorce the surveyor from the geodetic community? Not necessarily; a great deal depends upon the willingness of the profession to broaden its horizons. The new users of geodetic information require baseline information at accuracies comparable to and sometimes exceeding those the surveyor is accustomed to providing on a day-to-day basis. The surveyor needs to seek out his new customers and needs to become aware of his problems; he needs to upgrade his field operations, using the most precise instrumentation and adjustment techniques; and he most certainly must insist upon improving and expanding the university curriculum in surveying."

3. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time

Muneendra Kumar, Graduate Research Associate, part time

James P. Reilly, Graduate Research Associate, part time

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Susan Breslow, Research Aide, part time

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4. TRAVEL

Ivan I. Mueller
Bronx, New York, February 22, 1972
Attend GEOP Research Conference Steering Committee Meeting

Ivan I. Mueller
Graz, Austria, May 29-June 2, 1972
Present a paper at IAG International Symposium on Satellite and Terrestrial Triangulation

5. REPORTS PUBLISHED TO DATE

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by Hans D. Preuss
April, 1966

71 Proposed Optical Network for the National Geodetic Satellite Program
by Ivan I. Mueller
May, 1966

82 Preprocessing Optical Satellite Observations
by Frank D. Hotter
April, 1967

86 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 1 of 3: Formulation of Equations
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87 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 2 of 3: Computer Programs
by Edward J. Krakiwsky, George Blaha, Jack M. Ferrier
August, 1968

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93 Data Analysis in Connection with the National Geodetic Satellite Program
by Ivan I. Mueller
November, 1967

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100 Preprocessing Electronic Satellite Observations
by Joseph Gross
March, 1968

106 Comparison of Astrometric and Photogrammetric Plate Reduction Techniques for a Wild BC-4 Camera
by Daniel H. Hornbarger
March, 1968

110 Investigations into the Utilization of Passive Satellite Observational Data
by James P. Veach
June, 1968

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by Edward J. Krakiwsky
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118 The Use of Short Arc Orbital Constraints in the Adjustment of Geodetic Satellite Data
by Charles R. Schwarz
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by M.G. Arur
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August, 1970

147 Gravity Field Refinement by Satellite to Satellite Doppler Tracking
by Charles R. Schwarz
December, 1970

148 Inner Adjustment Constraints with Emphasis on Range Observations
by Georges Blaha
January, 1971

150 Investigations of Critical Configurations for Fundamental Range Networks
by Georges Blaha
March, 1971

177 Improvement of a Geodetic Triangulation through Control-Points Established by Means of Satellite or Precision Traversing
by Narendra K. Saxena
In press

The following papers were presented at various professional meetings:

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47th Annual meeting of the AGU, Washington, D.C., April 1966

"Preprocessing Optical Satellite Observational Data"
3rd Meeting of the Western European Satellite Subcommission, IAG, Venice, Italy, May 1967.

"Global Satellite Triangulation and Trilateration"
XIVth General Assembly of the IUGG, Lucerne, Switzerland, September 1967, (Bulletin Geodesique, March 1968).

"Investigations in Connection with the Geometric Analysis of Geodetic Satellite Data"
GEOS Program Review Meeting, Washington, D.C., Dec. 1967.

"Comparison of Photogrammetric and Astrometric Data Reduction Results for the Wild BC-4 Camera"
Conference on Photographic Astrometric Technique, Tampa, Fla., March 1968.

"Geodetic Utilization of Satellite Photography"
7th National Fall Meeting, AGU, San Francisco, Cal., Dec. 1968.

"Analyzing Passive-Satellite Photography for Geodetic Applications"
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"Sequential Least Squares Adjustment of Satellite Trilateration"
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"Experiments with SECOR Observations on GEOS-I"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"Experiments with Wild BC-4 Photographic Plates"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"Experiments with the Use of Orbital Constraints in the Case of Satellite Trails on Wild BC-4 Photographic Plates"
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Symposium on the Use of Artificial Satellites for Geodesy, Washington, D.C., April 15-17, 1971.

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"Scaling the SAO-69 Geometric Solution with C-Band Radar Data (Solution SC 11)"
Annual Fall Meeting of the American Geophysical Union, San Francisco, California, December 6-9, 1971.

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Annual Meeting of the Permanent Committee, International Federation of Surveyors, Tel Aviv, Israel, May 1972.

"Investigations on a Possible Improvement of Terrestrial Triangulation by Means of Super-Control Points"
IAG International Symposium - Satellite and Terrestrial Triangulation, Graz, Austria, June, 1972.

"Free Adjustment of a Geometric Global Satellite Network (Solution MPS7)"
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